M611 Fall 2019 Assignment 1, due Friday Sept. 6

1. [10 pts] Answer the following:

a. Show that for any positive integers $j \le k$

$$\binom{k}{j-1} + \binom{k}{j} = \binom{k+1}{j}.$$

b. Prove the Binomial Theorem

$$(a+b)^k = \sum_{j=0}^{k} \binom{k}{j} a^j b^{k-j},$$

for $k = 1, 2, \ldots$.

c. Does your proof work if a and b are square matrices that commute?

2. [10 pts] Prove the following theorem: If P and Q are commuting $n \times n$ matrices, then

$$e^P e^Q = e^{P+Q}.$$

Explain in your proof where the assumption that P and Q commute is used.

Note. You can use Theorem 3.51 from Rudin, stated here for matrices: If the series of $n \times n$ matrices $\sum_{k=0}^{\infty} A_k$, $\sum_{k=0}^{\infty} B_k$, and $\sum_{k=0}^{\infty} C_k$ converge respectively to the matrices $\mathbb{A}$, $\mathbb{B}$, and $\mathbb{C}$, and $C_k = \sum_{j=0}^{k} A_j B_{k-j}$, then $\mathbb{C} = \mathbb{A}\mathbb{B}$.

3. [10 pts] Solve the ODE system

$$\begin{aligned} y_1' &= y_1 + 4y_2; \quad y_1(0) = 0 \\ y_2' &= 5y_1 + 2y_2; \quad y_2(0) = 1. \end{aligned}$$

4. [10 pts] Solve the ODE system

$$\begin{aligned} y_1' &= y_1 - 2y_2; \quad y_1(0) = 0 \\ y_2' &= 3y_1 + y_2; \quad y_2(0) = 1. \end{aligned}$$

Express your solution without any complex values.