M612 Review for Midterm

The midterm exam will be Wednesday, March 4, 7:00-9:00 p.m., in Blocker 220. There will be four to six problems, depending on problem length, with some straightforward calculations and some proofs. **You will need to bring your own paper**. Class will not meet on Friday, March 6.

Possible topics include:

1. Fourier transforms

You should be able to compute the Fourier transform of a given function, solve linear constant coefficient PDE by taking a Fourier transform of the equation, and use Plancherel isometry to establish estimates in $L^2(\mathbb{R}^n)$ on solutions to linear constant coefficient PDE. See particularly Problems 2, 3, 4 in Assignment 1 and Problem 3 in Assignment 4.

1. Properties of Hölder and Sobolev spaces

You should know the definitions of $C^{k,\gamma}(\bar{U})$, $W^{k,p}(U)$, and $W_0^{k,p}(U)$. You should be able to check if a function is weakly differentiable, and you should be able to compute the regularity of a function space. See particularly Problems 1, 2, 3 in Assignment 3 and Problem 1 in Assignment 6.

2. Inequalities proved by approximation

In developing inequalities on Sobolev spaces, we often want to integrate by parts, and our standard trick for carrying this out is to approximate $u \in W^{k,p}(U)$ by functions in $C^\infty(\bar{U})$ or to approximate $u \in W_0^{k,p}(U)$ by functions in $C_c^\infty(U)$ and then take a limit after integration by parts has been carried out on the smooth functions. See particularly Problem 1 in Assignment 4 and Problems 3, 4 in Assignment 6.

3. Functional analysis

You should be clear on our definitions, including bounded, continuous, and compact operators; dual spaces; isometric isomorphisms and reflexive Banach spaces; weak convergence; compact, precompact, weakly compact, and weakly precompact sets; continuous and compact embeddings. See particularly Problems 1, 2, 3, 4 in Assignment 5.

4. Extension and Trace theorems

Understand the extension operator and the trace operator. See particularly Problem 2 in Assignment 6.

5. Embedding theorems

Know the upshot of Theorem 5.6.6. Please keep in mind that if a function has positive regularity we always associate it with its continuous version.