Elements of Set Theory, I

Note Title 6/5/2015

Let M be a set of elements, and let A, B be subsets of M. We define:

(i) $A \cup B = \{x \in M : x \in A \text{ or } x \in B\}$

(ii) $A \cap B = \{x \in M : x \in A \text{ and } x \in B\}$

(iii) $A \setminus B = \{x \in A : x \notin B\}$

(iv) $A^c = M \setminus A$

We write $A \subset B$ if every element of A is also an element of B.

Proposition 1 (De Morgan's Laws)

For any two sets A and B

(i) $(A \cup B)^c = A^c \cap B^c$

(ii) $(A \cap B)^c = A^c \cup B^c$.

Proof of (i)

We'll show $(A \cup B)^c \subset A^c \cap B^c$ and $A^c \cap B^c \subset (A \cup B)^c$.

For the first, let $x \in (A \cup B)^c$, which means precisely that x is in neither A nor B. I.e., $x \in A^c$ and $x \in B^c$. But this means $x \in A^c \cap B^c$.

To go the other way, suppose $y \in A^c \cap B^c$; which means $y \notin A$ and $y \notin B$, so that $y \notin A \cup B$. But this means $y \in (A \cup B)^c$.

$\square$