Note Title 6/6/2015

Elements of Set Theory, II

We can clearly extend De Morgan's laws to more than two sets. For example, consider $(A \cup B \cup C)^c$. We can view $B \cup C$ as a set so that

$$(A \cup (B \cup C))^c = A^c \cap (B \cup C)^c.$$

Then $(B \cup C)^c = B^c \cap C^c$ so

$$(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$$

More generally, if $\{A_k\}_{k=1}^n$ is any finite collection of sets, we have

$$\left(\bigcup_{k=1}^n A_k\right)^c = \bigcap_{k=1}^n A_k^c$$

and

$$\left(\bigcap_{k=1}^n A_k\right)^c = \bigcup_{k=1}^n A_k^c .$$

In fact, the same argument works for an infinite number of sets $\{A_k\}_{k=1}^\infty$, and we have

$$\left(\bigcup_{k=1}^{\infty} A_k\right)^c = \bigcap_{k=1}^{\infty} A_k^c$$

$$\left(\bigcap_{k=1}^{\infty} A_k\right)^c = \bigcup_{k=1}^{\infty} A_k^c.$$

Later, we'll see that indexing by integers can be restrictive, and we'll be interested in index sets such as $I = [0,1]$.

For $(A_i)_{i \in I}$ we have

$$\left(\bigcup_{i \in I} A_i \right)^c = \bigcap_{i \in I} A_i^c$$

$$\left(\bigcap_{i \in I} A_i \right)^c = \bigcup_{i \in I} A_i^c .$$