Note Title 7/29/2015

Ordinary Differential Equations

We consider first order ordinary differential equations (ODE) of the form

$$\frac{dy}{dx} = f(x,y); \qquad y(0) = y_0,$$

where $x \in \mathbb{R}$ and $y, f \in \mathbb{R}^n$.

We'll proceed by integrating this equation on $[0,x]$, giving:

$$y(x) - y_0 = \int_0^x f(t, y(t))\,dt$$

$$\Rightarrow \quad y(x) = y_0 + \int_0^x f(t, y(t))\,dt.$$

Let $y_1(x)$ denote an initial approximation for the solution, and suppose $y_1 \in C[0,b]$.

Then we can update y_1 with

$$y_2(x) = y_0 + \int_0^x f(t, y_1(t))\,dt$$

This leads to an iteration

$$y_{n+1}(x) = y_0 + \int_0^x f(t, y_n(t))\,dt,$$

which is called <u>Picard iteration</u>. Notice that if we set

$$T(y) = y_0 + \int_0^x f(t, y(t))\,dt,$$

then we again have a recursion of the form

$$y_{n+1} = T(y_n).$$