Note Title 8/11/2015

Equivalent Metrics on Compact Metric Spaces

Proposition 8.19

Suppose that (M, d) is compact, and that ρ is another metric on M. Then d and ρ are equivalent if and only if d and ρ are uniformly equivalent.

Proof

First, for $(\Longleftarrow)$ we notice that uniform equivalence always implies equivalence.

For $(\Rightarrow)$, the identity map $i:(M,d)\to(M,\rho)$ is continuous, and so i is a continuous function on a compact set, and so is uniformly continuous. By Theorem 8.4 $i(M)$ is compact, and since i is onto $(i(M)=M)$ this means (M,ρ) is compact. Since (M,ρ) is compact, we can repeat the argument above to see that i^{-1} is uniformly continuous on M. $\square$