Bernoulli's Inequality

Note Title 6/9/2015

Bernoulli's Inequality 1.7

If $a > -1$, $a \neq 0$, then

$$(1+a)^n > 1 + na$$

for any integer $n > 1$.

Proof

Proceed by induction, noting that the first applicable integer is $n = 2$. For $n = 2$, we have

$$(1+a)^2 = 1 + 2a + a^2 > 1 + 2a.$$

Now assume the inequality holds for $n > 1$, and let's check that it holds for $n+1$. We compute

$$(1+a)^{n+1} = (1+a)(1+a)^n > (1+a)(1+na)$$

$= 1 + (n+1)a + na^2 > 1 + (n+1)a,$

which is precisely Bernoulli's inequality for $n+1$. $\square$