Note Title 6/10/2015

Limits of Functions

Definition

Suppose $f(x)$ is a real-valued function defined in some open interval containing the point $a \in \mathbb{R}$, except possibly at a itself. We write

$$\lim_{x \to a} f(x) = L$$

provided that for every $\varepsilon > 0$ there exists $\delta > 0$ so that

$$0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon.$$

Carothers refers to this open interval with the value a omitted as a <u>punctured neighborhood</u> of a.

Theorem 1.17

Let f be a real-valued function defined in some punctured neighborhood of $a \in \mathbb{R}$. Then the following are equivalent:

(i) There exists a number $L \in \mathbb{R}$ so that

$$\lim_{x \to a} f(x) = L .$$

(ii) There exists a number $L \in \mathbb{R}$ so that

$$\lim_{n\to\infty} x_n = a \implies \lim_{n\to\infty} f(x_n) = L,$$

if $x_n \neq a \ \forall n$.

(iii) $\lim_{n\to\infty} x_n = a \implies \lim_{n\to\infty} f(x_n)$ converges to something if $x_n \neq a \ \forall n$.

The proof will be Problem 1.40 in HW.