Note Title 6/10/2015

Equivalence of $\mathbb{N}\times\mathbb{N}$ and $\mathbb{N}$

To see this, observe that every integer can be written as a power of 2 times an odd number. (Simply by factoring out the powers of 2.) For example,

$$24 = 2^3 \cdot 3, \quad 17 = 2^0 \cdot 17,$$

$$100 = 2^2 \cdot 25, \ \ldots$$

I.e.,

$$f(m,n) = 2^{m-1}(2n-1)$$

is a 1-1 correspondence between $\mathbb{N}\times\mathbb{N}$ and $\mathbb{N}$.

It will be more useful to us later to notice that there's another way to view this: we can arrange the elements of $\mathbb{N}\times\mathbb{N}$ and count them.

$$\begin{array}{ccccc}
\overset{1}{(1,1)} & \overset{3}{(1,2)} & \overset{6}{(1,3)} & \overset{10}{(1,4)} & \cdots \\
\overset{2}{(2,1)} & \overset{5}{(2,2)} & (2,3) & (2,4) & \cdots \\
\overset{4}{(3,1)} & \overset{8}{(3,2)} & (3,3) & (3,4) & \cdots \\
\overset{7}{(4,1)} & (4,2) & (4,3) & (4,4) & \cdots
\end{array}$$

etc.

Diagonalization Process.