Generalized Cantor Sets and 1-1

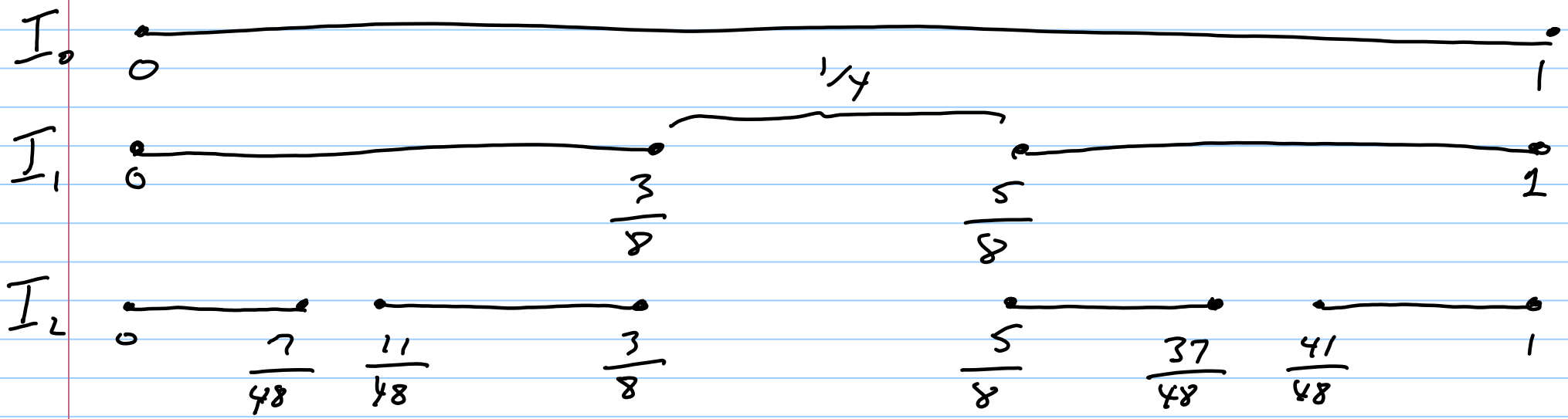
Note Title 6/14/2015

In constructing the Cantor set we deleted the middle third of each interval, but we could have deleted a different length such as a middle fourth.

A convenient choice to work with is to fix any $\alpha \in (0,1)$ and at step n to delete the middle intervals of length $\alpha 3^{-n}$.

For example, consider $\alpha = 3/4$.

$n=1$: I_1 we delete intervals of length $\frac{3}{4} \cdot 3^{-1} = 1/4$. For $n=2$: $\frac{3}{4} \cdot 3^{-2} = \frac{1}{12}$



We'll see in Problem 2.36 that the length of the resulting generalized Cantor set associated with α is $(1-\alpha)$.

Last, we consider a set defined in terms of the differences between elements of the Cantor set. I.e., for $x, y \in \Delta$ we consider values $y - x$. We denote

$$\Delta - \Delta := \{y - x : x, y \in \Delta\}.$$

For example, the Cantor set contains both $x = 1$ and $x = 0$, so

$$1 - 0 = 1 \in \Delta - \Delta$$

and $$0 - 1 = -1 \in \Delta - \Delta$$

We'll characterize $\Delta - \Delta$ more fully in the next lecture.