Note Title 7/15/2015

Interior and Closure

Definition (Interior)

The *interior* of a set A, which we'll denote $\operatorname{int}(A)$ or A°, is the largest open set contained in A. Precisely, we set

$$A^\circ = \bigcup \{ U : U \text{ is open, } U \subset A \}.$$

Recalling that each open set can be obtained as a union of balls, we see that

$$A^\circ = \bigcup \{ B_\varepsilon(x) : B_\varepsilon(x) \subset A \text{ for some } x \in A,\ \varepsilon > 0 \}$$

$$= \{ x \in A : B_\varepsilon(x) \subset A \text{ for some } \varepsilon > 0 \}.$$

Examples

1. For $A = [0,1]$, $A^\circ = (0,1)$

2. For $A = \{x : \|x\| \le r\}$, $A^\circ = \{x : \|x\| < r\}$.

Definition (Closure)

We define the <u>closure</u> of A, which we'll denote $\mathrm{cl}(A)$ or $\bar{A}$, to be the smallest closed set containing A. Precisely,

$$\bar{A} = \bigcap \{F : F \text{ is closed and } A \subset F\}.$$

Examples

1. For $A = (0,1)$, $\bar{A} = [0,1]$

2. $\overline{\mathbb{Q}} = \mathbb{R}$

3. $\overline{\Delta} = \Delta$ (i.e., the Cantor set is closed).