Note on Topology

Note Title 7/18/2015

The subject topology is often described as the study of open sets. More precisely, Suppose τ is a collection of subsets of M. We say τ is a <u>topology</u> if:

(i) $\emptyset, M \in \tau$

(ii) Any union of elements of τ is an element of τ.

(iii) Any intersection of finitely many elements of τ is an element of τ.

Notice that the set τ of all open subsets of $\mathbb{R}$, $\mathbb{R}^n$, or any other metric space is a topology.

We've seen that the open sets on a space can be described by the continuous functions on the space, so we could just as well say topology is the study of continuous functions.

If a property is determined entirely by the open sets of a space (or equivalently, the continuous functions) we say it's a <u>topological property</u>. For example, <u>separability</u> (see Problem 4.48) is a topological property, while boundedness of a set is not.

Since homeomorphisms preserve the structure of open sets (map open sets to open sets) topological properties are preserved by homeomorphisms.