Space-Filling Curves

Note Title 7/23/2015

We've seen that no continuous 1-1 and onto map can exist between $[0,1]$ and $[0,1] \times [0,1]$ (otherwise they would be homeomorphic). Perhaps surprisingly, it is possible to construct a continuous map from $[0,1]$ to $[0,1] \times [0,1]$ that's onto. Such a map is an example of a space-filling curve.

We can describe a map from $[0,1]$ to $[0,1]\times[0,1]$ as

$$f(t) = (x(t), y(t));$$

i.e., a typical parametrized curve.

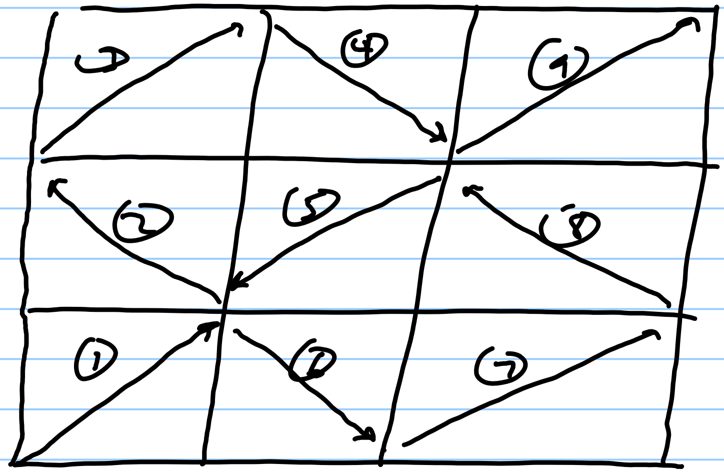
Peano suggested that we could proceed by constructing a sequence of maps $f_1, f_2, \ldots$

and taking $n \to \infty$. He constructed his sequence as follows. For f_1, he divided $[0,1] \times [0,1]$ into 9 squares, started in the lower left corner and traced out a path to the upper right corner:

$[0,1]\times[0,1]$

$(1,1)$

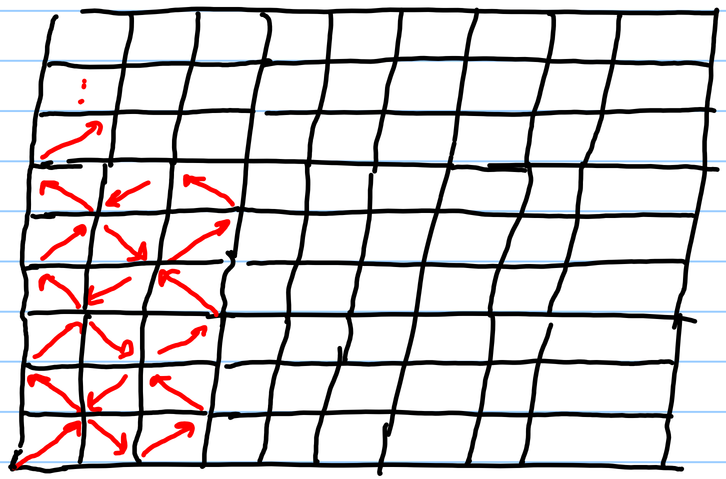
f_1 :



$(0,0)$

$[0,1]\times[0,1]$

f_2 :



It looks plausible that as $n\to\infty$ this path will cross every point in $[0,1]\times[0,1]$, and in fact it's possible to verify this (though we won't).