Note Title 7/26/2015

Subsets of Complete Metric Spaces

Theorem 7.9

Let (M, d) be a complete metric space and let A be a subset of M. Then (A, d) is complete iff A is closed in M.

Proof

First, for $(\Rightarrow)$ assume (A, d) is complete, and take $(x_n) \subset A$ so that $x_n \to x$ in M. But (x_n) is Cauchy (since it converges) so it must converge to $x \in A$ (since A is assumed complete).

For $(\Leftarrow)$ suppose A is closed in M and let $(x_n) \subset A$ be Cauchy. Since M is complete (x_n) converges in M, and since A is closed we must have its limit $x \in A$. $\square$