M642 Assignment 3, due Friday Feb. 8

1. (**Keener Problem 4.2.1.**) Construct a Green's function for

$$u'' = f(x); \quad u(0) = u'(1) = 0,$$

and express the solution in terms of this Green's function.

Note. Cf. Problem 3.4.2d.

2. (**Keener Problem 4.2.3.**) Construct a Green's function for

$$u'' + \alpha^2 u = f(x); \quad u(0) = u(1), u'(0) = u'(1),$$

and express the solution in terms of this Green's function. For what values of α does the Green's function fail to exist?

3. (**Keener Problem 4.2.6.**) Construct the Green's function for

$$\begin{aligned} u'' + \frac{3}{2x}u' - \frac{3}{2}\frac{1}{x^2}u &= f(x) \\ u(0) &= 0 \\ u'(1) &= 0. \end{aligned}$$

Express the solution in terms of the Green's function.

4. (**Keener Problem 4.2.8.**) The operator $Lu = u'' + 4u$ with boundary conditions $u'(0) = u'(\pi)$, $u(0) = u(\pi)$ has no Green's function. Why?

5. (**Keener Problem 4.2.9.**) Convert the differential equation

$$u'' + \lambda u = f(x), \quad u(0) = \alpha, u(1) = \beta$$

to a Fredholm integral equation of the form

$$u(x) = \lambda \int k(x, \xi) u(\xi) d\xi + g(x).$$