M642 Assignment 6, due Friday March 1

1. [10 pts] (**Keener Problem 4.4.2.**) Find the modified Green's function for

$$\begin{aligned} u'' &= f(x) \\ u(0) + u(1) &= 0 \\ u'(0) - u'(1) &= 0. \end{aligned}$$

2. [10 pts] (**Keener Problem 4.4.3.**) Find the modified Green's function for

$$\begin{aligned} u'' + 4\pi^2 u &= f(x) \\ u(0) &= u(1) \\ u'(0) &= u'(1). \end{aligned}$$

Note. Judging by the solution, Keener means for the problem to be stated with the sign here, and not with the sign he actually states (in front of $4\pi^2$).

3. [10 pts] (**Keener Problem 4.4.7.**) Solve the following equation in the least squares sense

$$\begin{aligned} u'' &= \sin^2 x \\ u'(0) &= \alpha \\ u'(\pi) &= \beta. \end{aligned}$$

4. [10 pts] (**Keener Problem 4.4.8.**) Solve the following problem in the best possible (least squares) sense:

$$\begin{aligned} u'' + u &= \sin^3 x \\ u(0) &= 1 \\ u(\pi) &= 2. \end{aligned}$$

5. [10 pts] Consider the fourth order eigenvalue problem

$$\begin{aligned} u'''' - \lambda u &= 0 \\ u(0) = 0; \quad u(L) &= 0 \\ u''(0) = 0; \quad u''(L) &= 0, \end{aligned}$$

for some constant $L > 0$.

a. Show that there are no negative eigenvalues for this problem, and that $\lambda = 0$ is not an eigenvalue.

b. Find the eigenvalues and eigenfunctions for this problem.