M647 Assignment 7, due Friday March 22

1. [10 pts] Let $\alpha > 1$ and consider the equation

$$\begin{aligned} -u'' &= f \\ u(0) &= 0 \\ \alpha u(1) + u'(1) &= 0. \end{aligned}$$

a. Show that with these boundary conditions $Lu = -u''$ is positive definite.

b. Use the method of eigenfunction expansion to solve this equation. You need not identify your eigenvalues algebraically, but indicate them graphically. Also, determine their approximate values as k gets large. (Expressing them as $\{\lambda_k\}_{k=1}^{\infty}$, ordered so that $\lambda_1 < \lambda_2 < \dots$).

2. [10 pts] (**Keener Problem 4.5.1.**) Use the method of eigenfunction expansion to solve the equation

$$\begin{aligned} u'' &= f(x) \\ u'(0) &= \alpha \\ u'(1) &= \beta. \end{aligned}$$

Note. Omit the part that Keener assigns as a numerical calculation.

3. [10 pts] (**Keener Problem 4.5.2.**) Use the method of eigenfunction expansion to solve

$$\begin{aligned} u'' + u &= f(x) \\ u(0) &= u(2\pi) \\ u'(0) &= u'(2\pi). \end{aligned}$$

Note. Clearly, you should expand a bit on Keener's hint.

4. [10 pts] Solve the quadratic equation

$$z^2 - 3z + 3 + i = 0,$$

expressing your solutions in the form $a + ib$.

5. [10 pts] Compute the following values:

a. $\sqrt{4}$ with branch $[0, 2\pi)$.

b. $\sqrt{4}$ with branch $(0, 2\pi]$.

c. $\ln(-i)$ with branch $[0, 2\pi)$.

d. $\ln(1 + i)$ with branch $[0, 2\pi)$.

e. i^i with branch $[-\pi, \pi)$.

f. $(-i)^{1+i}$ with branch $[-\pi, \pi)$.