MATH 151, Spring 2021
COMMON EXAM 1 - ONLINE EXAM VERSION A

The 5 workout problems make up 43 points of the exam and the 19 multiple choice problems make up 57 points (3 points each), for a total of 100 points. **No calculator is allowed!**

PART I: WORK OUT PROBLEMS

<u>Directions</u>: Present each of your solutions on an empty sheet/side of paper. *Show all of your work* neatly and concisely and *box your final answer.* You will be graded not merely on the final answer, but also the quality and correctness of the work leading up to it.

1. (8 pts) Use the **definition** of the derivative to find $f'(x)$ for $f(x) = \sqrt{8-x}$. *No shortcuts are allowed!*

2. (8 pts) Consider the function $g(x) = \begin{cases} 3x^2 + A, & \text{if } x < -2, \\ Bx - 7, & \text{if } x = -2, \\ Ax + 3, & \text{if } x > -2. \end{cases}$

   Find the values of A and B that will make $g(x)$ continuous. If no such values exist, then explain why.

3. (7 pts) A pilot steers a plane in the direction $210°$ counterclockwise from the positive x-axis at a speed of 400 mph. The wind is blowing in a direction $60°$ counterclockwise from the positive x-axis at a speed of 20 mph. Find the true (resultant) velocity **vector** of the plane. (Your answer should be a vector.)

4. (14 pts) Evaluate these limits. Do not use the L'Hôpital method.

   (a) $\lim\limits_{x \to 7^+} \dfrac{x^2 - 4x - 21}{|7 - x|}$
   (b) $\lim\limits_{x \to 3} \dfrac{\frac{1}{x} - \frac{1}{3}}{3x - x^2}$
   (c) $\lim\limits_{x \to -4} \dfrac{x + 4}{\sqrt{x^2 + 9} - 5}$

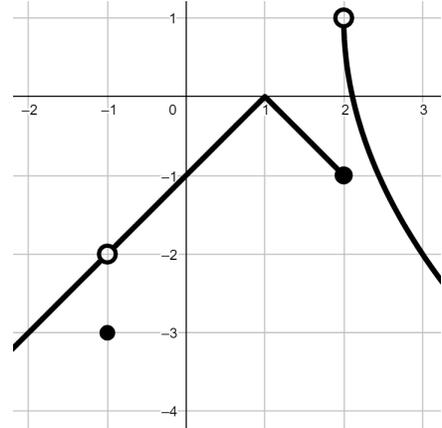5. (6 pts) Consider the function $f(x) = \begin{cases} 16 - x^2, & \text{if } x \leq -1, \\ 8 - 5x, & \text{if } -1 < x \leq 3, \\ x^3 + 1, & \text{if } x > 3. \end{cases}$

   (a) Evaluate $\lim\limits_{x \to 3} f(x)$, or explain why it does not exist.

   (b) Evaluate $\lim\limits_{x \to -1^+} f(x)$, or explain why it does not exist.

Multiple choice problems begin on the next page...

PART II: Multiple Choice. 3 points each

6. The following is the graph of a function $y = f(x)$. Which of the following statements concerning the graph is **TRUE**?

   (a) $f(2) = 1$

   (b) $f(x)$ is continuous at $x = -1$

   (c) $f(x)$ is continuous from the left at $x = 2$

   (d) $\displaystyle\lim_{x \to 2^+} f(x) = -1$

   (e) $\displaystyle\lim_{x \to -1} f(x)$ does not exist

7. The displacement (in feet) of a particle moving in a straight line is given by $s(t) = t^2 + 5t - 6$, where t is measured in seconds. Find the average velocity over the interval $[1, 4]$.

   (a) $\dfrac{15}{2}$ ft/s

   (b) 30 ft/s

   (c) $\dfrac{10}{3}$ ft/s

   (d) 0 ft/s

   (e) 10 ft/s

8. Given $\mathbf{a} = \langle 2, -1 \rangle$ and $\mathbf{b} = \langle 1, -3 \rangle$, find the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$.

   (a) $\dfrac{5}{\sqrt{5}}$

   (b) $\dfrac{1}{2}\mathbf{i} - \dfrac{3}{2}\mathbf{j}$

   (c) 1

   (d) $2\mathbf{i} - \mathbf{j}$

   (e) $\dfrac{5}{\sqrt{10}}$

9. Which of the following are parametric equations of a line passing through the point $(3, -7)$ and perpendicular to the vector $\langle 6, -5 \rangle$?

   (a) $x = -7 - 6t, y = 3 + 5t$

   (b) $x = 3 + 5t, y = -7 + 6t$

   (c) $x = 3 - 6t, y = -7 + 5t$

   (d) $x = -7 + 5t, y = 3 + 6t$

   (e) $x = 3 - 5t, y = -7 + 6t$

10. Evaluate $\lim\limits_{x \to 3} \dfrac{2x^2 + 6x}{x^2 - 2x - 15}$.

    (a) 3

    (b) ∞

    (c) -3

    (d) $-\infty$

    (e) $-\dfrac{3}{2}$

11. Evaluate $\lim\limits_{x \to -\infty} \dfrac{8x + 3}{\sqrt{5x + 16x^2}}$.

    (a) -2

    (b) $\dfrac{8}{\sqrt{5}}$

    (c) $\dfrac{1}{2}$

    (d) $-\dfrac{1}{2}$

    (e) 2

12. Find the point of intersection of the following two lines, if it exists.

$$L_1 : \mathbf{r}(t) = \langle 7 - 3t, 4 + t \rangle$$
$$L_2 : \mathbf{r}(s) = \langle 1 + s, 2 + s \rangle$$

    (a) the lines do not interest

    (b) $(1, 3)$

    (c) $(2, 5)$

    (d) $(3, 4)$

    (e) $(4, 5)$

13. Simplify $\sin\left(\arccos\left(\dfrac{x}{3}\right)\right)$, $-3 < x < 3$, to an algebraic expression.

    (a) $\dfrac{\sqrt{9 - x^2}}{3}$

    (b) $\dfrac{x}{\sqrt{9 - x^2}}$

    (c) $\dfrac{\sqrt{9 + x^2}}{x}$

    (d) $\dfrac{3}{\sqrt{9 - x^2}}$

    (e) $\dfrac{\sqrt{9 - x^2}}{x}$

14. Evaluate $\displaystyle\lim_{x\to-\infty} \arctan\left(\frac{6x + x^3}{2x - 5x^2}\right)$.

   (a) ∞

   (b) 0

   (c) $-\infty$

   (d) $\dfrac{\pi}{2}$

   (e) $-\dfrac{\pi}{2}$

15. Which of the following statements is true regarding the equation $4 = x^5 + 2x$?

   (a) A solution exists on the interval $(1, 2)$ by the Squeeze Theorem

   (b) A solution exists on the interval $(1, 2)$ by the Intermediate Value Theorem

   (c) A solution exists on the interval $(0, 1)$ by the Squeeze Theorem

   (d) A solution exists on the interval $(0, 1)$ by the Intermediate Value Theorem

   (e) the equation has no real number solutions

16. Find the cosine of the angle between the vectors $\langle -4, 2\rangle$ and $\langle 1, 5\rangle$.

   (a) $\dfrac{6}{\sqrt{20}\sqrt{26}}$

   (b) $-\dfrac{40}{\sqrt{20}}$

   (c) $\dfrac{6}{\sqrt{26}}$

   (d) $-\dfrac{40}{\sqrt{20}\sqrt{26}}$

   (e) $\dfrac{6}{\sqrt{20}}$

17. A sled is pulled along a level path by a rope. A 30-lb force acting an angle $60°$ above the horizontal moves the sled 6 ft. Find the work done by the force.

   (a) $90\sqrt{3}$ J

   (b) 180 J

   (c) $90\sqrt{2}$ J

   (d) 45 J

   (e) 90 J

18. Evaluate $\displaystyle\lim_{x\to\infty} \frac{7e^{6x} - 5e^{-3x}}{4e^{6x} + 2e^{-3x}}$.

   (a) $-\dfrac{5}{2}$

   (b) 0

   (c) $\dfrac{7}{4}$

   (d) ∞

   (e) $-\infty$

19. Find the distance between the point $(-2, 5)$ to the line $y = \dfrac{1}{3}x + 1$.

(a) $\dfrac{17}{\sqrt{29}}$

(b) $\dfrac{14}{\sqrt{10}}$

(c) $\dfrac{2}{\sqrt{10}}$

(d) $\dfrac{14}{\sqrt{20}}$

(e) $\dfrac{2}{\sqrt{20}}$

20. Find all vertical asymptotes of the function $f(x) = \dfrac{x^2 - 9}{x^3 + x^2 - 12x}$

(a) $x = 0$, $x = -4$, and $x = 3$

(b) $x = 0$ and $x = -4$

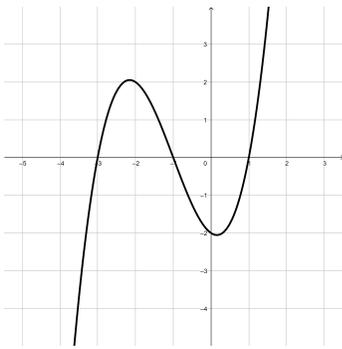(c) $x = 0$ and $x = 3$

(d) $x = -4$ and $x = 3$

(e) $x = -4$ only

21. Find a vector of length 4 in the same direction as the vector from the point $(3, -5)$ and $(-2, 7)$.

(a) $\langle -20, 48 \rangle$

(b) $\left\langle -\dfrac{20}{\sqrt{13}}, \dfrac{48}{\sqrt{13}} \right\rangle$

(c) $\left\langle -\dfrac{5}{4}, 3 \right\rangle$

(d) $\left\langle -\dfrac{20}{13}, \dfrac{48}{13} \right\rangle$

(e) $\left\langle \dfrac{20}{13}, -\dfrac{48}{13} \right\rangle$

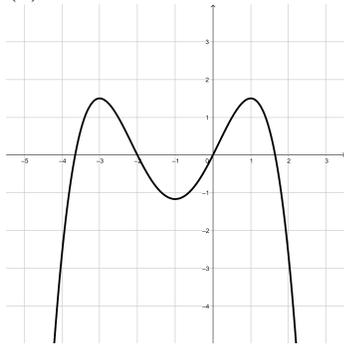22. Evaluate $\displaystyle\lim_{x \to -5^+} \dfrac{2 - x}{x^2 + 4x - 5}$.

(a) $-\infty$

(b) -1

(c) 0

(d) $-\dfrac{2}{5}$

(e) ∞
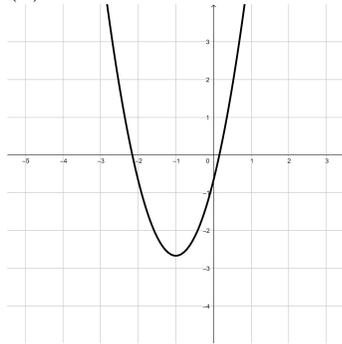
23. The following is the graph of graph of $f'(x)$.

Which of the following is the graph of $f(x)$?
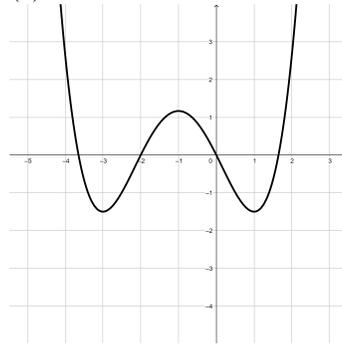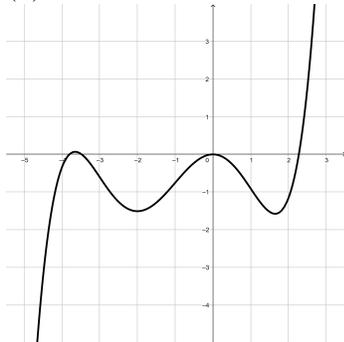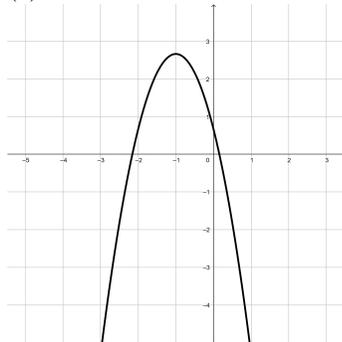
(a)

(b)

(c)

(d)

(e)

24. Given $f(x) = x^3 - 3x + 1$ and $f'(x) = 3x^2 - 3$, find the equation of the tangent line to $f(x)$ at $x = -2$.

(a) $y = 3x - 1$

(b) $y = 3x + 17$

(c) $y = 9x - 1$

(d) $y = 3x + 5$

(e) $y = 9x + 17$