MATH 151, SPRING 2022
COMMON EXAM III - VERSION A

LAST NAME(print): ____________________ FIRST NAME(print): ____________________

INSTRUCTOR: ____________________

SECTION NUMBER: __________

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam on the ScanTron form.***

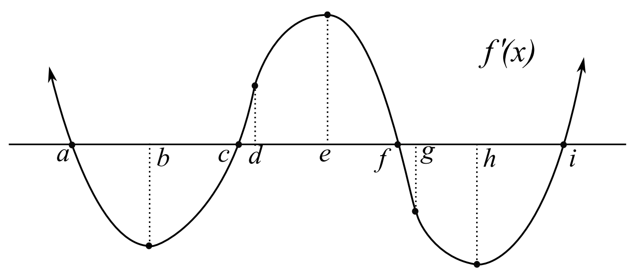
THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________________

PART I: Multiple Choice. 4 points each

The graph below is the graph of the DERIVATIVE of some function f. Use the graph to answer Questions 1 and 2.



1. For what values of x does f have a local minimum?

   (a) a and f
   (b) b and h
   (c) c and i $\leftarrow$ correct
   (d) d
   (e) g

2. On what interval(s) is f concave down?

   (a) $(-\infty, d) \cup (g, \infty)$
   (b) $(-\infty, b) \cup (e, h)$ $\leftarrow$ correct
   (c) $(b, e) \cup (h, \infty)$
   (d) $(a, c) \cup (f, i)$
   (e) (d, g) only

3. Given that $f(x)$ is defined everywhere except $x = -3$ and $f'(x) = \dfrac{x^2(1-x)}{(x+3)^3}$, on what intervals is $f(x)$ increasing?

   (a) $(-\infty, 0) \cup (1, \infty)$
   (b) $(-\infty, -3) \cup (1, \infty)$
   (c) $(-\infty, 0) \cup (0, 1)$
   (d) $(-3, 1)$ $\leftarrow$ correct
   (e) $(-3, 0) \cup (1, \infty)$

4. Consider the function $f(x) = 3x^4 - 8x^3 + 5$. At what value(s) of x does $f(x)$ have a local minimum?

   (a) $x = 0$ and $x = 2$

   (b) $x = 0$

   (c) $x = \frac{3}{4}$

   (d) $x = 2$ $\leftarrow$ correct

   (e) f has no local minima.

5. Find the absolute extrema for $f(x) = 3x - x^3$ on $[0, 3]$

   (a) Absolute maximum is 0, Absolute minimum is -18

   (b) Absolute maximum is 2, Absolute minimum is -2

   (c) Absolute maximum is 2, Absolute minimum is 0

   (d) Absolute maximum is 4, Absolute minimum is -2

   (e) Absolute maximum is 2, Absolute minimum is -18 $\leftarrow$ correct

6. Find the value(s) of c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^2 - 5x + 3$ on the interval $[1, 3]$.

   (a) $c = -1$

   (b) $c = 1$

   (c) $c = 2$ $\leftarrow$ correct

   (d) $c = 3$

   (e) This function does not satisfy the condition of The Mean Value Theorem on the given interval.

7. Compute $\lim_{x \to 0} \dfrac{\sin(x) - x + 4x^2}{x^3 + 3x^2}$

(a) $\dfrac{1}{6}$

(b) 0

(c) $-\dfrac{1}{6}$

(d) $\dfrac{4}{3}$ $\leftarrow$ correct

(e) $\dfrac{3}{2}$

8. The domain of $f(x)$ is all real numbers and $f''(x) = x^2(x^2+1)(x-3)$. Find the $x-$coordinate(s) of all inflection points for the function $f(x)$.

(a) $x = 3$ $\leftarrow$ correct

(b) $x = 0, 3$

(c) $x = 1, -1, 3$

(d) $x = 0, 1, -1, 3$

(e) f has no inflection points

9. An object is traveling at a speed of 60 m/s when the brakes are fully applied, producing a constant deceleration of 12 meters per second squared. What is the distance covered before the object comes to a stop?

(a) 150 m $\leftarrow$ correct

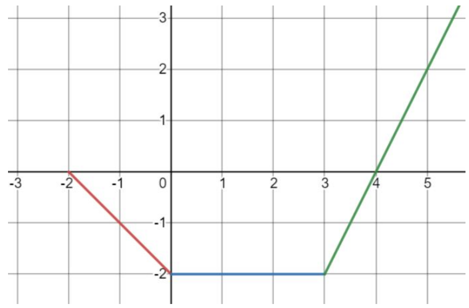
(b) 200 m

(c) 210 m

(d) 310 m

(e) 450 m

10. The following is the graph of f. Evaluate $\int_{-2}^{5} f(x)dx$



(a) -10
(b) -9
(c) -8 $\leftarrow$ correct
(d) 8
(e) 9

11. Suppose f'' is continuous on $(-\infty, \infty)$. If $f'(0) = 0$ and $f''(0) = 1$, then what can you say about f?

(a) At $x = 0$, f has a local minimum $\leftarrow$ correct
(b) At $x = 0$, f has a local maximum
(c) At $x = 0$, f has neither a maximum nor a minimum
(d) At $x = 0$, f has an inflection point
(e) More information is needed to determine if f has a maximum or minimum at $x = 0$

12. Find an antiderivative $F(x)$ of $f(x) = \dfrac{x^3 - x^2 + 5}{x^4}$.

(a) $F(x) = \ln|x| + \dfrac{1}{x} - \dfrac{1}{x^5} + C$
(b) $F(x) = \ln|x| + \dfrac{1}{x} - \dfrac{5}{3x^3} + C$ $\leftarrow$ correct
(c) $F(x) = \ln|x| + \dfrac{1}{3x^3} - \dfrac{5}{3x^3} + C$
(d) $F(x) = \ln|x| + \dfrac{1}{3x^3} - \dfrac{1}{x^5} + C$
(e) None of the above

13. A particle is moving with acceleration $a(t) = 3\sin(t) - 6t$, $v(0) = 5$, and $s(0) = 7$. Find the position function $s(t)$ for the particle.

(a) $s(t) = 3\sin(t) - t^3 + 8t + 7$

(b) $s(t) = 3\sin(t) - t^3 + 5t + 7$

(c) $s(t) = -3\sin(t) - t^3 + 8t + 7$ $\leftarrow$ correct

(d) $s(t) = -3\sin(t) - t^3 + 5t + 7$

(e) $s(t) = -3\sin(t) - t^3 + 2t + 7$

14. Which of the following gives the exact area under the curve $f(x) = \ln(x)$ on the interval $[1, 7]$? Assume the right endpoint is used.

(a) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{6}{n} \ln\left(\frac{6}{n}i\right)$

(b) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{7}{n} \ln\left(1 + \frac{6}{n}i\right)$

(c) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{7}{n} \ln\left(\frac{7}{n}i\right)$

(d) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{7}{n} \ln\left(\frac{7}{n}i\right)$

(e) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{6}{n} \ln\left(1 + \frac{6}{n}i\right)$ $\leftarrow$ correct

15. Estimate the area under the graph of $f(x) = x^2 + 2$ from $x = -3$ to $x = 6$ using three rectangles of equal width and left endpoints.

(a) 54

(b) 72 $\leftarrow$ correct

(c) 153

(d) 186

(e) 125

16. Given that $\int_5^1 f(x)\,dx = 3$, $\int_1^3 g(x)\,dx = 1$, $\int_3^5 g(x)\,dx = 2$ determine the value of $\int_5^1 \big(f(x) - g(x)\big)\,dx$.

(a) 6 $\leftarrow$ correct

(b) 3

(c) 2

(d) 0

(e) -6

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

17. (7 points) Find the most general antiderivative for the function.

$f'(x) = \sec^2 x + \dfrac{3}{x} + \dfrac{1}{\sqrt[5]{x^4}} + \dfrac{3}{1+x^2}$

Sol. The most general antiderivative of f' is

$$f(x) = \tan x + 3\ln|x| + 5x^{1/5} + 3\arctan x + C$$

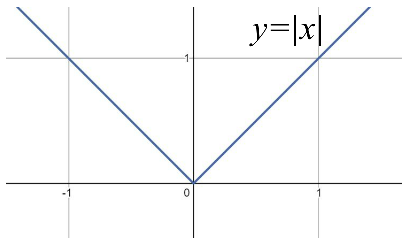
18. (7 points) Evaluate the integral by interpreting it in terms of areas.

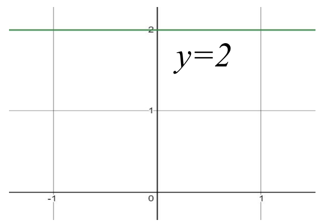
$$\int_{-1}^{1} \Big(|x| + 2\Big)\ dx$$

Sol. By the linearity, we have

$$\int_{-1}^{1} \Big(|x| + 2\Big)\ dx = \int_{-1}^{1} |x|\ dx + \int_{-1}^{1} 2\ dx$$

The first integral represents the area of the region determined by $y = |x|$ on $[-1, 1]$. The second represents the area of the rectangle with base 2 and height 2. See the figure below.

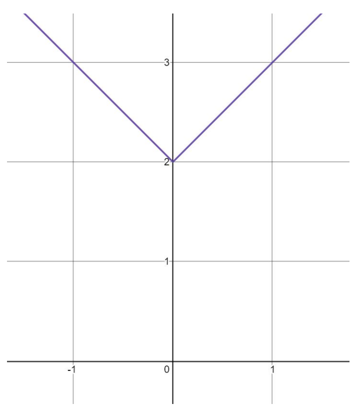




Thus, the integral becomes

$$\begin{aligned}\int_{-1}^{1} \Big(|x| + 2\Big)\ dx =& \int_{-1}^{1} |x|\ dx + \int_{-1}^{1} 2\ dx \\ =& 2 \cdot \frac{1}{2} \cdot 1 \cdot 1 + 2 \cdot 2 = 5.\end{aligned}$$

Alternatively, the graph of $y = |x| + 2$ is the vertical shift of $y = |x|$ by 2. Thus the given integral represents the aref of the region consisting of 2 identical trapezoids with height 1, base 3, and top 2.



Thus, the integral becomes

$$\int_{-1}^{1} \Big(|x| + 2\Big)\ dx = 2 \cdot \frac{1(3 + 2)}{2} = 5.$$

19. (10 points) Find $\lim_{x\to 0}\left(1+3x\right)^{\frac{1}{x}}$

Sol. Take the natural log of $y=\left(1+3x\right)^{\frac{1}{x}}$ to have

$$\ln y=\ln\left(1+3x\right)^{\frac{1}{x}}=\frac{1}{x}\ln\left(1+3x\right).$$

Consider the limit

$$\lim_{x\to 0}\ln y=\lim_{x\to 0}\frac{\ln\left(1+3x\right)}{x}.$$

Since the limit is indeterminate form of $\frac{0}{0}$ as $x\to 0$, we can apply L'Hospital's Rule.

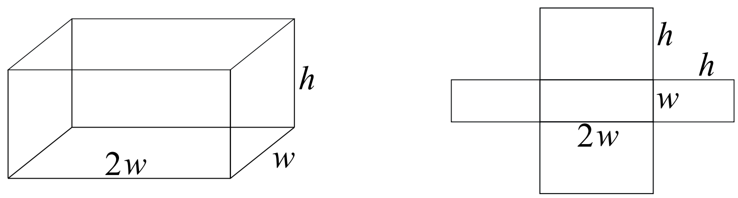
$$\lim_{x\to 0}\ln y=\lim_{x\to 0}\frac{\ln\left(1+3x\right)}{x}\overset{\text{L'H}}{=}\lim_{x\to 0}\frac{\frac{3}{1+3x}}{1}=\frac{\frac{3}{1+0}}{1}=3.$$

Since the exponential function e^x is continuous, the limit of composition becomes

$$\lim_{x\to 0}\left(1+3x\right)^{\frac{1}{x}}=\lim_{x\to 0}y=\lim_{x\to 0}e^{\ln y}=e^{\left(\lim_{x\to 0}\ln y\right)}=e^3$$

20. (12 points) A rectangular box with an open top is to have a volume of 8m^3. The length of this base is twice the width. Material for the base costs \$3 per square meter. Material for the sides costs \$4 per square meter. Determine the width of the box that minimizes the cost of the container. Be sure to show that your answer is a minimum.

Sol. Let w and h be the width and the height of the base respectively. From the condition, we can labe the edges of the box as in the figure.



Since the volume of the box is 8m^3, we have

$$2w \cdot w \cdot h = 8 \quad \Rightarrow \quad h = \frac{4}{w^2}$$

The cost can be expressed as

$$3 \cdot 2w \cdot w + 4 \cdot \Big(2 \cdot wh + 2 \cdot 2wh\Big) = 6w^2 + 4 \cdot 6wh = 6w^2 + 4 \cdot 6w\Big(\frac{4}{w^2}\Big)$$

which yields the cost function $C(w)$

$$C(w) = 6w^2 + \frac{4 \cdot 6 \cdot 4}{w}$$

The above cost function has the derivative

$$C'(w) = 12w - \frac{4 \cdot 6 \cdot 4}{w^2}$$

So $C'(w) = 0$ only when $w = 2$ because

$$12w = \frac{4 \cdot 6 \cdot 4}{w^2} \quad \Leftrightarrow \quad w^3 = \frac{4 \cdot 6 \cdot 4}{12} = 8 = 2^3.$$

We claim that the cost function $C(w)$ attains the minimum at $w = 2$. Since

$$C''(w) = 12 + 2\frac{4 \cdot 6 \cdot 4}{w^3} > 0 \text{ for all } w > 0$$

the cost function $C(w)$ is concave upward, and so the only critical number $w = 2$ determines the only local minimum, and hence the absolute minimum of $C(w)$ by the 2^{nd} Derivative Test. We can also use the 1^{st} Derivative Test. One can check that $C'(w)$ changes its sign at $w = 2$. For example,

$$C'(1) = 12 - 4 \cdot 6 \cdot 4 < 0 \text{ and } C'(4) = 12 - \frac{4 \cdot 6 \cdot 4}{4^2} = 12 - 6 > 0.$$

The desired width of the box is 2 m.