MATH151, Fall 2022
Common Exam II - Version B

LAST NAME (print): ________________________ FIRST NAME (print): ________________________

INSTRUCTOR: ______________________ UIN: ________________________

SECTION NUMBER: ____________

DIRECTIONS:

- No calculators, cell phones, smart watches, headphones, or other electronic devices may be used, and must be put away.
- TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
- In Part I, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
- In Part II, present your solutions in the space provided. Show all your work neatly and concisely, and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- **Be sure to fill in your name, UIN, section number, and version letter of the exam on the ScanTron form.**

THE AGGIE HONOR CODE

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____

Part I: Multiple Choice. 3.5 points each

1. The position of a particle moving in a straight line is given by $s(t) = \frac{2}{3}t^3 - t^2 + 5t$, where s is in feet and t is in seconds. Find the velocity at the time where the acceleration of the particle is 10 ft/s^2.

   (a) 3 ft/s
   (b) 9 ft/s
   (c) 17 ft/s
   (d) 24 ft/s
   (e) 5 ft/s

2. Find the slope of the tangent line to $f(x) = \arctan(2x)$ at $x = 3$.

   (a) $\dfrac{2}{37}$
   (b) $\dfrac{2}{19}$
   (c) $\dfrac{1}{37}$
   (d) $\dfrac{1}{19}$
   (e) $\dfrac{1}{7}$

3. A bacteria culture grows at a rate proportional to its size. It has 100 bacteria initially and 400 bacteria after 3 hours. How long will it take for there to be 1000 bacteria?

   (a) $\dfrac{4\ln(10)}{\ln(3)}$ hours
   (b) $\dfrac{\ln(10)}{3\ln(4)}$ hours
   (c) $\dfrac{30}{\ln(4)}$ hours
   (d) $\dfrac{3\ln(10)}{\ln(4)}$ hours
   (e) None of these.

Suppose f and g are differentiable functions. Use the following table of values to answer questions 4 and 5.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	3	4	2	1
1	-1	2	4	-5

4. Find $h'(-1)$ if $h(x) = [f(x^2)]^2$.

   (a) 4

   (b) -8

   (c) -24

   (d) 16

   (e) Not enough information.

5. Find $H'(1)$ if $H(x) = \dfrac{g(f(x))}{2x+1}$.

   (a) $\dfrac{20}{9}$

   (b) $\dfrac{4}{3}$

   (c) $-\dfrac{64}{9}$

   (d) $-\dfrac{2}{9}$

   (e) $\dfrac{4}{9}$

6. Find the linearization of $f(x) = x^{3/2}$ at $a = 4$ and use it to approximate $f(4.1)$.

(a) 7.9
(b) 8.1
(c) 8.2
(d) 8.3
(e) 8.4

7. Find the slope of the tangent line to the curve $3y - xy^2 = 18$ at the point $(-3, 2)$.

(a) $\dfrac{22}{15}$
(b) $-\dfrac{4}{9}$
(c) $-\dfrac{22}{9}$
(d) $-\dfrac{8}{3}$
(e) $\dfrac{4}{15}$

8. Find a tangent vector to the curve $\mathbf{r}(t) = \langle \sqrt{6-t},\, te^{3t} \rangle$ at the point where $t = 2$.

(a) $\left\langle -\frac{1}{4},\, 7e^6 \right\rangle$
(b) $\left\langle -\frac{1}{4},\, 3e^6 \right\rangle$
(c) $\left\langle \frac{1}{4},\, 7e^6 \right\rangle$
(d) $\left\langle \frac{1}{4},\, 3e^6 \right\rangle$
(e) $\langle -1,\, e^6 \rangle$

9. Find the 123rd derivative of $f(x) = \frac{1}{3}\cos(3x)$.

   (a) $3^{122}\cos(3x)$
   (b) $-3^{122}\sin(3x)$
   (c) $3^{122}\sin(3x)$
   (d) $-3^{123}\cos(3x)$
   (e) $3^{123}\sin(3x)$

10. An object is in motion according to $s(t) = t^2 - 4t + 7$, $t \geq 0$, where t is in seconds and s is in feet. Find the total distance traveled by the object during the first three seconds.

   (a) 2 ft
   (b) 3 ft
   (c) 4 ft
   (d) 5 ft
   (e) 7 ft

11. The radius of a sphere was measured to be 10 cm with a maximum error in measurement of 0.2 cm. Use differentials to estimate the maximum error in the calculated surface area of the sphere. (The surface area of a sphere is $S = 4\pi r^2$.)

   (a) 4π cm^2
   (b) 16π cm^2
   (c) 8π cm^2
   (d) 40π cm^2
   (e) 80π cm^2

12. Which of the following is true?

$$f(x) = \begin{cases} 2x^3 + 5 & \text{if } x < 1 \\ 3x^2 + 4 & \text{if } 1 \le x \le 3 \\ 10x + 1 & \text{if } x > 3 \end{cases}$$

(a) f is continuous but not differentiable at $x = 1$; f is differentiable at $x = 3$

(b) f is continuous but not differentiable at both $x = 1$ and $x = 3$

(c) f is differentiable at both $x = 1$ and $x = 3$

(d) f is differentiable at $x = 1$; f is continuous but not differentiable at $x = 3$

(e) f is discontinuous at both $x = 1$ and $x = 3$

13. There are two lines tangent to the parabola $f(x) = x^2 + 1$ that pass through the point $(1, -2)$. Find the x-coordinates where these tangent lines touch the parabola.

(a) $x = -1, 3$

(b) $x = 0, 2$

(c) $x = -2, 4$

(d) $x = -\frac{1}{2}, \frac{5}{2}$

(e) None of these.

14. Find the t-value(s) where the curve $x = \sin^2 t + \cos t$, $y = t - \cos t$, $0 \le t < 2\pi$ has a vertical tangent line.

(a) $\frac{\pi}{3}, \frac{5\pi}{3}$

(b) $0, \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

(c) $0, \frac{\pi}{6}, \frac{11\pi}{6}, \pi$

(d) $\frac{3\pi}{2}$

(e) $0, \frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \pi$

15. Find the derivative of $f(x) = \ln\left(\frac{\csc^3(x)}{\sqrt{x^4+5}}\right)$.

(a) $3\csc(x) - \frac{1}{8x^3}$

(b) $-3\cot(x) - \frac{4x^3}{x^4+5}$

(c) $-3\cot(x) - \frac{2x^3}{x^4+5}$

(d) $-\frac{3}{\sec(x)\tan(x)} - \frac{2x^3}{x^4+5}$

(e) $-3\csc^2(x)\cot(x) - \frac{4x^3}{\sqrt{x^4+5}}$

16. Find the slope of the tangent line to the curve $x = 6t^2 - 26$, $y = t^2 + 3t + 2$ at the point $(-2, 0)$.

(a) $\frac{7}{24}$

(b) $\frac{1}{24}$

(c) $-\frac{1}{24}$

(d) $\frac{24}{7}$

(e) 24

Part II: Work Out Problems

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

17. (15 points) Differentiate the following functions. Do not simplify after taking the derivative.

(a) $y = \sec^5\left(2x^4 + 5^{-3x}\right)$

(b) $y = \dfrac{\tan(2x)}{(x^4 + e^{\pi})^2}$

(c) $y = \log_8(2 - x^2)e^{\sqrt{x}}$

18. (8 points) Differentiate $y = (x^4 + 5x^3)^{\arcsin(3x)}$. Your answer should be in terms of x only.

19. (9 points) Find $\dfrac{dy}{dx}$ for $\cos(xy^2) + e^{x+y} = 5$.

20. (12 points) A camera is positioned 6 m away from the launching point of a weather balloon. The balloon is released and rises vertically at a speed of 4 m/s.

(a) At what rate is the distance between the camera and the balloon changing when the balloon has risen 8 m?

(b) If the camera is always kept aimed at the balloon, how fast is the camera's angle of elevation changing when the balloon has risen 8 m?

Do not write in this table.

Question	Points Awarded	Points
1–16		56
17		15
18		8
19		9
20		12
Total		100