MATH151, Fall 2022
Common Exam III - Version A

LAST NAME (print): ________________________ FIRST NAME (print): ________________________

INSTRUCTOR: ________________________ UIN: ________________________

SECTION NUMBER: _____________

DIRECTIONS:

- No calculators, cell phones, smart watches, headphones, or other electronic devices may be used, and must be put away.
- TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
- In Part I, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
- In Part II, present your solutions in the space provided. Show all your work neatly and concisely, and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- **Be sure to fill in your name, UIN, section number, and version letter of the exam on the ScanTron form.**

THE AGGIE HONOR CODE

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____

Part I: Multiple Choice. 4 points each

1. Suppose $f(x)$ has a domain of all real numbers and $f''(x) = x^3(x-2)^5(x^2+6x-16)$. Find the x-coordinates of the inflection points of f.

   (a) $x = -8$ only

   (b) $x = 0$ only

   (c) $x = -8$ and $x = 0$

   (d) $x = -8$, $x = 0$, and $x = 2$

   (e) $x = 0$ and $x = 2$

2. Find $\lim_{x \to 0} \dfrac{e^{3x} - 3x - \cos(x)}{2x^3 + x^2}$.

   (a) 5

   (b) 0

   (c) ∞

   (d) 2

   (e) $\dfrac{7}{2}$

3. Find the value(s) of c that satisfy the conclusion of the Mean Value Theorem for the $f(x) = x^3 - x$ on the interval $[0, 3]$.

   (a) $\sqrt{\dfrac{7}{3}}$

   (b) $\dfrac{3}{2}$

   (c) 1

   (d) $\sqrt{\dfrac{10}{3}}$

   (e) $\sqrt{3}$

4. Find the absolute maximum and minimum values of $f(x) = 6\sqrt{x} - x + 1$ on $[0, 25]$.

(a) Absolute maximum is 10; absolute minimum is 1

(b) Absolute maximum is 10; absolute minimum is 6

(c) Absolute maximum is 9; absolute minimum is 0

(d) Absolute maximum is 6; absolute minimum is 1

(e) Absolute maximum is 10; absolute minimum is 0

5. Set up the limit to find the area under the graph of $f(x) = \sqrt{x+3}$ on $[1, 4]$.

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{3 + \frac{3i}{n}}$

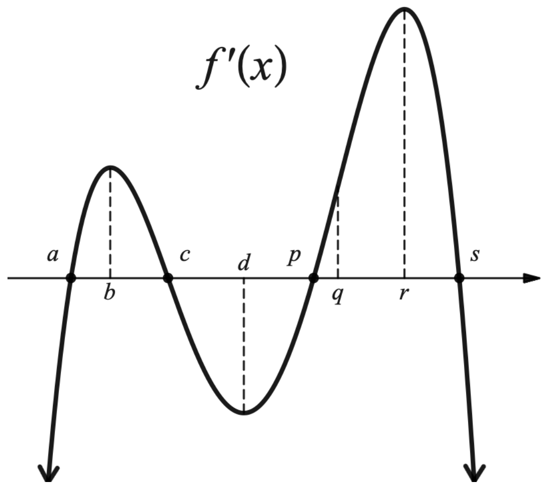
(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1 + \frac{3i}{n}}$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\sqrt{4 + \frac{4i}{n}}$

(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{4 + \frac{3i}{n}}$

(e) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\sqrt{3 + \frac{4i}{n}}$

The graph below is the **derivative**, $f'(x)$, of a continuous function f whose domain is all real numbers. Use this graph to answer questions 6 and 7.



6. Find the value(s) of x where f has a local maximum.

   (a) $x = a$ and $x = p$
   (b) $x = c$ and $x = s$
   (c) $x = b$ and $x = r$
   (d) $x = d$
   (e) Cannot be determined.

7. Find the interval(s) where f is concave up.

   (a) (b, d), (r, ∞)
   (b) (a, c), (p, s)
   (c) $(-\infty, b)$, (d, r)
   (d) (c, q)
   (e) (c, p)

8. Given $\int_{-1}^{2} f(x)\,dx = 1$, $\int_{2}^{5} f(x)\,dx = 4$ and $\int_{5}^{-1} g(x)\,dx = -5$, find $\int_{-1}^{5} [f(x) + 3g(x)]\,dx$.

(a) 20
(b) -10
(c) 10
(d) 19
(e) -11

9. Find the interval(s) where $g(x) = e^x(x^2 - x - 5)$ is decreasing.

(a) $(-\infty, -3)$, $(2, \infty)$
(b) $(-3, 0)$, $(0, 2)$
(c) $(-\infty, -3)$, $(0, \infty)$
(d) $(-3, 2)$
(e) $(-3, \infty)$

10. Suppose $f'(x) = \dfrac{2x + \sqrt[3]{x^4} - \sqrt{x}}{x}$. Find $f(0)$ if $f(1) = 2$.

(a) $\dfrac{13}{4}$
(b) $\dfrac{2}{3}$
(c) $\dfrac{8}{3}$
(d) $\dfrac{5}{4}$
(e) 0

11. Estimate the area under the graph of $f(x) = 25 - x^2$ from $x = -2$ to $x = 4$ using 3 rectangles of equal width and midpoints.

(a) 132
(b) 134
(c) 128
(d) 110
(e) 192

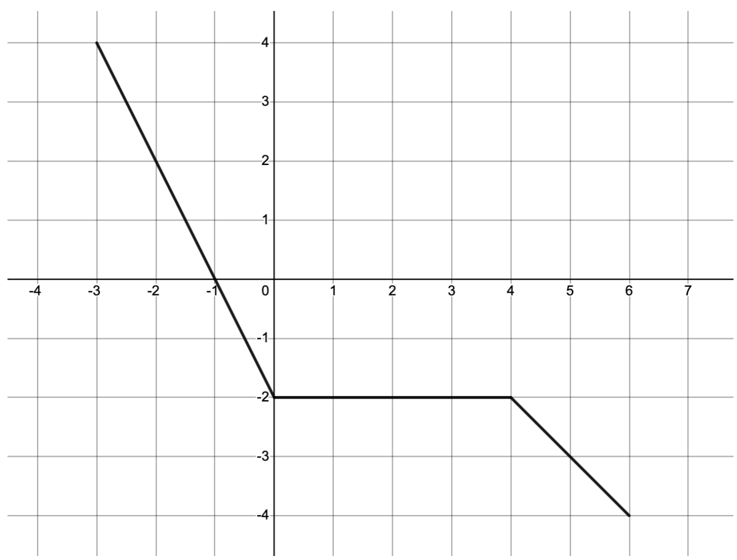
12. Which of the following is **true**?

(a) Every function attains an absolute minimum and absolute maximum on a closed interval.
(b) A left Riemann sum is always an underestimate.
(c) Any limit of the form $\infty - \infty$ evaluates to 0.
(d) $G(x) = \arccos x$ is the only antiderivative of $g(x) = \dfrac{-1}{\sqrt{1 - x^2}}$.
(e) If $f'(-3) = 0$ and $f''(-3) = 4$, then f has a local minimum at $x = -3$.

13. Find $\lim_{x\to\infty} x \tan\left(\frac{5}{x}\right)$.

(a) 5
(b) 0
(c) 1
(d) ∞
(e) $\frac{1}{5}$

14. Use the graph of f below to evaluate $\int_{-3}^{6} f(x)\,dx$.



(a) -7
(b) -11
(c) -16
(d) 19
(e) -10

15. An object is traveling at 10 m/s when it starts to accelerate at 6 m/s^2. How far does the object travel before reaching a speed of 40 m/s?

(a) 125 m
(b) 50 m
(c) 150 m
(d) 110 m
(e) 200 m

16. Find the critical numbers of $f(x) = x^{2/5}(x-6)^2$.

(a) $x = 1, 6$
(b) $x = 0, 6$
(c) $x = 6, 12$
(d) $x = 0, 1, 6$
(e) $x = 0, 6, 12$

Part II: Work Out Problems

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

17. (6 points) Find the most general antiderivative of $f'(x) = \sec x \tan x + \dfrac{5}{1+x^2} + 3^x + \dfrac{6}{x} + \sin x$.

18. (9 points) Find $\lim\limits_{x \to 0^+} (\cos x)^{3/x^2}$.

19. (10 points) Consider the function $f(x) = \dfrac{x+1}{(x-3)^2}$, for which $f'(x) = \dfrac{-x-5}{(x-3)^3}$ and $f''(x) = \dfrac{2x+18}{(x-3)^4}$.

(a) What is the domain of f?

domain: ______________________

(b) Determine the interval(s) on which f is increasing or decreasing. If there are none, write DNE.

increasing: ______________________ decreasing: ______________________

(c) Determine the x-coordinates of any local extrema. If there are none, write DNE.

local maximum at $x =$ ________________ local minimum at $x =$ ________________

(d) Determine the interval(s) on which f is concave upward or concave downward. If there are none, write DNE.

concave up: ______________________ concave down: ______________________

(e) Determine the x-coordinates of any inflection points. If there are none, write DNE.

inflection point at $x =$ ________________

20. (11 points) The top and bottom margins of a poster are each 2 in and the side margins are 1 in. The poster is to have a total area of 128 in^2. Find the dimensions of the poster that will maximize the printed area. Justify that your answer gives a maximum.

Do not write in this table.

Question	Points Awarded	Points
1–16		64
17		6
18		9
19		10
20		11
Total		100