Exam2 version B

Wednesday, November 3, 2021 10:29 AM

exam2Bkey

 Fall 2021

MATH 152, FALL 2021
COMMON EXAM II - VERSION B KEY

LAST NAME(print): ____________ FIRST NAME(print): ____________

INSTRUCTOR: ____________

SECTION NUMBER: ____________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2 (Problems 16-19), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*
6. **Again. The use of a calculator, laptop or computer is prohibited.**

THE AGGIE HONOR CODE

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: ____________

[illegible]

(e) $9\int \cos^2\theta\, d\theta$ key

$= \int \sqrt{9-9\sin^2\theta} \cdots$

$= \sqrt{9(1-\sin^2\theta)} = \sqrt{9\cos^2\theta}$

$= \int 3\cos\theta \cdot 3\cos\theta \cdot d\theta$

$= 9\int \cos^2\theta\, d\theta$

10. The integral $\int_0^\infty e^{-2x}\, dx$

(a) converges to $\frac{1}{4}$
(b) converges to $\frac{1}{2}$ key
(c) converges to 0
(d) converges to 2
(e) diverges

$= \lim_{t\to\infty} \int_0^t e^{-2x}\, dx = \lim_{t\to\infty} \left[-\frac{1}{2} e^{-2x} \Big|_0^t \right] = -\frac{1}{2}\lim_{t\to\infty} \left[\left(\frac{1}{e^{2t}}\right) - \frac{1}{e^0} \right] = -\frac{1}{2}\left(-\frac{1}{1}\right) = \frac{1}{2}$

11. Use the Remainder Estimate for the Integral Test to determine the minimum number of terms needed to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ to within $\frac{1}{80}$.

(a) 5 terms
(b) 8 terms
(c) 6 terms
(d) 4 terms
(e) 7 terms key

want $R_n \le \int_n^\infty \frac{1}{x^3}\, dx < \frac{1}{80}$

$= -\frac{1}{2x^2}\Big|_n^\infty = 0 - \left(-\frac{1}{2n^2}\right) = \frac{1}{2n^2} < \frac{1}{80}$

$\Rightarrow 80 < 2n^2$
$\Rightarrow 40 < n^2$
$\therefore$ the smallest n is 7.

12. Which statement is true about the integral $\int_0^4 \frac{2}{(x-3)^2}\, dx$?

(a) Converges to $\frac{8}{3}$
(b) Converges to $-\frac{8}{3}$
(c) Converges to $\frac{4}{3}$
(d) Converges to $-\frac{4}{3}$
(e) Diverges key

$= \lim_{t\to 3^-} \int_0^t \frac{2}{(x-3)^2}\, dx + \lim_{s\to 3^+} \int_s^4 \frac{2}{(x-3)^2}\, dx$

$= \lim_{t\to 3^-} \left[-\frac{2}{x-3} \Big|_0^t \right] + \lim_{s\to 3^+} \left[-\frac{2}{x-3} \Big|_s^4 \right]$

$= \left(-\frac{2}{0^-}\right) - \left(-\frac{2}{-3}\right) + \left(-\frac{2}{1}\right) - \left(-\frac{2}{0^+}\right)$

$= +\infty - \frac{2}{3} - 2 + \infty = \infty$: diverges.

13. Which of the following integrals is equivalent to $\int \sqrt{4x^2-9}\, dx$?

(a) $2\int \sec\theta \tan^2\theta\, d\theta$
(b) $2\int \sec^2\theta \tan\theta\, d\theta$
(c) $\frac{9}{2}\int \tan\theta\, d\theta$
(d) $\frac{9}{2}\int \sec^2\theta \tan\theta\, d\theta$
(e) $\frac{9}{2}\int \sec\theta \tan^2\theta\, d\theta$ key

Let $x = \frac{3}{2}\sec\theta$
$\Rightarrow dx = \frac{3}{2}\sec\theta \cdot \tan\theta \cdot d\theta$

Then
$= \int \sqrt{9\sec^2\theta - 9} \cdot \frac{3}{2}\sec\theta \cdot \tan\theta \cdot d\theta$

$\sqrt{9(\sec^2\theta - 1)} = \sqrt{9 \cdot \tan^2\theta}$

$= \int 3\tan\theta \cdot \frac{3}{2}\sec\theta \cdot \tan\theta\, d\theta$

$= \frac{9}{2}\int \sec\theta \cdot \tan^2\theta\, d\theta$

14. Write out the form of the partial fraction decomposition of the function

$$f(x) = \frac{x^3 - 2x^2 - 5x + 4}{(x+2)^2(x^2-1)(x^2+5x+7)}$$

(a) $\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1} + \frac{D}{x+1} + \frac{Ex+F}{x^2+5x+7}$ key
(b) $\frac{Ax+B}{(x+2)^2} + \frac{Cx+D}{x^2-1} + \frac{Ex+F}{x^2+5x+7}$
(c) $\frac{A}{(x+2)^2} + \frac{B}{x-1} + \frac{C}{x+1} + \frac{Dx+E}{x^2+5x+7}$
(d) $\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx+D}{x^2-1} + \frac{Ex+F}{x^2+5x+7}$
(e) $\frac{A}{(x+2)^2} + \frac{B}{x^2-1} + \frac{Cx+D}{x^2+5x+7}$

$= \frac{x^3-2x^2-5x+4}{(x+2)^2 \cdot (x-1)(x+1)(x^2+5x+7)}$

15. Consider the series $\sum_{n=1}^{\infty} a_n$ whose n-th partial sum is given by $s_n = \frac{2}{3-e^{-2n}}$. What is $\sum_{n=1}^{\infty} a_n$?

(a) 2
(b) 1
(c) $\frac{1}{3}$
(d) $\frac{2}{3}$ key
(e) 0

$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} \sum_{k=1}^{n} a_k = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{2}{3 - e^{-2n}} = \frac{2}{3}$

Part 2: Work Out

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (12 pts) Evaluate $\int \frac{1}{x^2\sqrt{x^2+4}}\, dx$.

key: $-\frac{\sqrt{x^2+4}}{4x} + C$

Let $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta \cdot d\theta$

$= \int \frac{1}{4\tan^2\theta \sqrt{4\tan^2\theta + 4}} \cdot 2\sec^2\theta \cdot d\theta$

$\sqrt{4(\tan^2\theta+1)} = \sqrt{4\sec^2\theta}$

$= \int \frac{1}{4\tan^2\theta \cdot 2\sec\theta} \cdot 2\sec^2\theta \cdot d\theta = \frac{1}{4}\int \frac{1}{\tan^2\theta} \cdot \sec\theta\, d\theta$

$= \frac{1}{4}\int \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\cos\theta}\, d\theta = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta} \cdot d\theta$

Let $u = \sin\theta$
$\Rightarrow du = \cos\theta \cdot d\theta$

$= \frac{1}{4}\int \frac{1}{u^2} \cdot d\theta = -\frac{1}{4u} + C$

$= -\frac{1}{4\sin\theta} + C$

And since $x = 2\tan\theta \Rightarrow \tan\theta = \frac{x}{2}$

$\Rightarrow$ (triangle with sides x, 2, hypotenuse $\sqrt{x^2+4}$) $\Rightarrow \sin\theta = \frac{x}{\sqrt{x^2+4}}$

$\therefore = -\frac{\sqrt{x^2+4}}{4x} + C$

17. (8 pts) Consider the following series

$$\sum_{n=1}^{\infty} \frac{1-3^{n-1}}{3^{2n}}$$

(a) Determine whether the series coverges or diverges, and state the reason.

(b) If it converges, find its sum. If it diverges, write DIVERGES.

key: $-\frac{1}{24}$

$\sum_{n=1}^{\infty} \frac{1-3^{n-1}}{3^{2n}} = \sum_{n=1}^{\infty} \frac{1}{3^{2n}} - \sum_{n=1}^{\infty} \frac{3^{n-1}}{3^{2n}}$

$= \sum_{n=1}^{\infty} \frac{1}{(3^2)^n} - \frac{1}{3}\sum_{n=1}^{\infty} \frac{3^n}{3^{2n}}$

$= \sum_{n=1}^{\infty} \left(\frac{1}{9}\right)^n - \frac{1}{3}\sum \left(\frac{3}{3^2}\right)^n$: geometric ($= r_1$, $= r_2$)

Since $|r_1| = \frac{1}{9} < 1$ and $|r_2| = \frac{1}{3} < 1$, the series converges. } answer for part a)

And its sum is

$= \frac{\frac{1}{9}}{1-\frac{1}{9}} - \frac{1}{3} \cdot \frac{\frac{1}{3}}{1-\frac{1}{3}}$

$= \frac{\frac{1}{9}}{\frac{8}{9}} - \frac{1}{3} \cdot \frac{\frac{1}{3}}{\frac{2}{3}}$

$= \frac{1}{8} - \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{8} - \frac{1}{6} = -\frac{1}{24}$

18. (12 pts) Evaluate $\int \frac{-2x+4}{(x^2+1)(x+1)}\, dx$

key: $-\frac{3}{2}\ln(x^2+1) + \arctan x + 3\ln|x+1| + C$

$\frac{-2x+4}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$

$\Rightarrow -2x+4 = (Ax+B)(x+1) + C(x^2+1)$

- $x=-1$: $6 = 0 + C \cdot (2)$ $\therefore C = 3$
- $x=0$: $4 = B + 3$ $\therefore B = 1$
- $x=1$: $2 = (A+1)(2) + 3 \cdot (2)$
  $\Rightarrow -4 = (A+1)(2)$ $\therefore A = -3$

Then

$= \int \frac{-3x+1}{x^2+1} + \frac{3}{x+1}\, dx$

$= -3\int \frac{x}{x^2+1}\, dx + \int \frac{1}{x^2+1}\, dx + 3\int \frac{1}{x+1}\, dx$

$= -\frac{3}{2}\ln(x^2+1) + \arctan x + 3\ln|x+1| + C$

19. (8 pts) Use the Integral Test to determine whether $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges or diverges. Support your answer.

key: $\int_2^\infty \frac{1}{x(\ln x)^2}\, dx = \frac{1}{\ln 2}$. *Since the improper integral converges, so the series converges by the Integral Test.*

Let $f(x) = \frac{1}{x(\ln x)^2}$: continuous, positive, and decreasing in $[2,\infty)$

Then the integral test:

$\int_2^\infty \frac{1}{x(\ln x)^2}\, dx = \lim_{t\to\infty} \int_2^t \frac{1}{x(\ln x)^2}\, dx$

Let $u = \ln x \Rightarrow du = \frac{1}{x}\, dx$

$= \lim_{t\to\infty} \int_\square^\triangle \frac{1}{u^2}\, du$

$= \lim_{t\to\infty} \left[-\frac{1}{u} \Big|_\square^\triangle \right]$

$= \lim_{t\to\infty} \left[-\frac{1}{\ln x} \Big|_2^t \right]$

$= \lim_{t\to\infty} \left[\left(-\frac{1}{\ln t}\right) - \left(-\frac{1}{\ln 2}\right) \right]$

$= \frac{1}{\ln 2}$: converges

Since the improper integral converges, so the series converges by the Integral Test.