

**MATH 152, FALL 2021**  
**COMMON EXAM III - VERSION B**

LAST NAME(print): \_\_\_\_\_ FIRST NAME(print): \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

SECTION NUMBER: \_\_\_\_\_

**DIRECTIONS:**

1. The use of a calculator, laptop or computer is prohibited.
2. **TURN OFF** cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2 (Problems 16-19), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form*.
6. **Again. The use of a calculator, laptop or computer is prohibited.**

THE AGGIE HONOR CODE

**“An Aggie does not lie, cheat, or steal, or tolerate those who do.”**

Signature: \_\_\_\_\_

**FOR INSTRUCTOR USE ONLY**

Question	Points Awarded	Points
1-15	ScanTron	60
16		8
17		8
18		12
19		12
<b>TOTAL</b>		100

Part 1: Multiple Choice (4 points each)

1. Which of the following statements is true for the series  $\sum_{n=1}^{\infty} \frac{3 + \sin n}{n^5 + 1}$ ?

- (a) The series converges since  $\frac{3 + \sin n}{n^5 + 1} < \frac{3}{n^5}$  and  $\sum_{n=1}^{\infty} \frac{3}{n^5}$  converges.
- (b) The series converges since  $\frac{3 + \sin n}{n^5 + 1} < \frac{4}{n^5}$  and  $\sum_{n=1}^{\infty} \frac{4}{n^5}$  converges.
- (c) The series converges since  $\frac{3 + \sin n}{n^5 + 1} > \frac{2}{n^5}$  and  $\sum_{n=1}^{\infty} \frac{2}{n^5}$  converges.
- (d) The series diverges since  $\frac{3 + \sin n}{n^5 + 1} > \frac{2}{n^5}$  and  $\sum_{n=1}^{\infty} \frac{2}{n^5}$  diverges.
- (e) None of these.

2. For which series is the ratio test inconclusive?

- (a)  $\sum_{n=1}^{\infty} \frac{n+2}{n!}$
- (b)  $\sum_{n=2}^{\infty} \frac{(-1)^n}{3^n \sqrt{\ln n}}$
- (c)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$
- (d)  $\sum_{n=1}^{\infty} \frac{n}{2^n}$
- (e)  $\sum_{n=1}^{\infty} n e^{-n}$

3. Which of the following statements is true for the series  $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2 + 1}$ ?

- (a) The series converges absolutely.
- (b) The series converges but not absolutely.
- (c) The series diverges by the test for divergence.
- (d) The series diverges by the alternating series test.
- (e) None of these.

4. Find the coefficient of  $x^3$  in the Maclaurin series for the function  $f(x) = \sin(2x)$ .

- (a)  $\frac{2}{3}$
- (b)  $-\frac{2}{3}$
- (c)  $\frac{4}{3}$
- (d)  $-\frac{4}{3}$
- (e)  $\frac{1}{3}$

5. The series  $\sum_{n=2}^{\infty} c_n x^n$  converges when  $x = 4$  and diverges when  $x = -7$ . What can be said about the convergence of the following series?

$$(I) \sum_{n=2}^{\infty} c_n 9^n \qquad (II) \sum_{n=2}^{\infty} c_n (-4)^n$$

- (a) (I) diverges, (II) is inconclusive.
- (b) (I) diverges, (II) converges.
- (c) (I) is inconclusive, (II) converges.
- (d) Both (I) and (II) converge.
- (e) Both (I) and (II) are inconclusive.

6. Which of the following is true regarding the series  $\sum_{n=1}^{\infty} \frac{5n \cdot 3^n}{4^n}$ .

- (a) The Ratio Test limit is  $\frac{15}{4}$ , so the series diverges.
- (b) The Ratio Test limit is  $\frac{15}{4}$ , so the series converges.
- (c) The Ratio Test limit is  $\frac{9}{4}$ , so the series diverges.
- (d) The Ratio Test limit is  $\frac{3}{4}$ , so the series diverges.
- (e) The Ratio Test limit is  $\frac{3}{4}$ , so the series converges.

7. Find the Maclaurin series for the function  $f(x) = x^2 e^{-x^3}$ .

(a)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+2}}{n!}$

(b)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+6}}{n!}$

(c)  $\sum_{n=0}^{\infty} \frac{x^{3n+6}}{n!}$

(d)  $\sum_{n=0}^{\infty} \frac{x^{3n+2}}{n!}$

(e)  $\sum_{n=0}^{\infty} \frac{x^{5n}}{n!}$

8. Find the sum of the series  $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{4^{2n} (2n)!}$

(a)  $\cos 3$

(b)  $\cos\left(\frac{3}{4}\right)$

(c)  $3 \cos\left(\frac{3}{4}\right)$

(d)  $\sin\left(\frac{3}{4}\right)$

(e)  $\sin 3$

9. Find the interval of convergence of the series  $\sum_{n=0}^{\infty} \frac{n!(x+4)^n}{3^n}$ .

(a)  $(-\infty, \infty)$

(b)  $(-7, -1)$

(c)  $(-4, 4)$

(d)  $\{-4\}$

(e)  $\{0\}$

10. Find a power series representation for  $f(x) = \frac{x}{x+4}$  and its radius of convergence.

(a)  $\sum_{n=0}^{\infty} \frac{x^{n+1}}{4^{n+1}}, R = 4$

(b)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{4^{n+1}}, R = 4$

(c)  $\sum_{n=0}^{\infty} \frac{x^{n+1}}{4^{n+1}}, R = \frac{1}{4}$

(d)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{4^{n+1}}, R = \frac{1}{4}$

(e)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{4^{n+1}}, R = 4$

11. Suppose that  $0 < a_n < b_n$  for all  $n \geq 1$ . Which of the following statements is always true?

(a) If  $\sum_{n=1}^{\infty} b_n$  is divergent, then so is  $\sum_{n=1}^{\infty} a_n$ .

(b) If  $\sum_{n=1}^{\infty} a_n$  is convergent, then so is  $\sum_{n=1}^{\infty} b_n$ .

(c) If  $\lim_{n \rightarrow \infty} b_n = 0$ , then  $\sum_{n=1}^{\infty} a_n$  is convergent.

(d) If  $\lim_{n \rightarrow \infty} a_n = 0$ , then  $\lim_{n \rightarrow \infty} b_n = 0$ .

(e) If  $\sum_{n=1}^{\infty} a_n$  is divergent, then so is  $\sum_{n=1}^{\infty} b_n$ .

12. Find the 3rd degree Taylor polynomial,  $T_3(x)$ , for the function  $f(x) = \ln x$  centered at  $a = 6$ .

(a)  $T_3(x) = \ln 6 + \frac{1}{6}(x-6) - \frac{1}{72}(x-6)^2 + \frac{1}{648}(x-6)^3$

(b)  $T_3(x) = \ln 6 + \frac{1}{6}(x-6) - \frac{1}{36}(x-6)^2 + \frac{1}{108}(x-6)^3$

(c)  $T_3(x) = \ln 6 + \frac{1}{6}(x-6) - \frac{1}{6}(x-6)^2 + \frac{1}{36}(x-6)^3$

(d)  $T_3(x) = \ln 6 + \frac{1}{6}(x-6) - \frac{1}{36}(x-6)^2 + \frac{1}{216}(x-6)^3$

(e)  $T_3(x) = \ln 6 + \frac{1}{6}(x-6) - \frac{1}{6}(x-6)^2 + \frac{1}{12}(x-6)^3$

13. The alternating series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+2)^2}$  converges. Use the Alternating Series Estimation Theorem to determine an upper bound on the absolute value of the error in using  $s_5$  to approximate the sum of the series.

- (a)  $\frac{1}{8}$
- (b)  $\frac{1}{9}$
- (c)  $\frac{1}{35}$
- (d)  $\frac{1}{64}$
- (e)  $\frac{1}{81}$

14. Consider the series below, which statement is true regarding the absolute convergence of each series?

$$(I) \sum_{n=1}^{\infty} \frac{(-2)^n}{3^{n+1}} \qquad (II) \sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 3}$$

- (a) (I) converges but not absolutely, (II) converges absolutely.
- (b) (I) converges absolutely, (II) converges but not absolutely.
- (c) Both (I) and (II) converge but not absolutely.
- (d) Both (I) and (II) converges absolutely.
- (e) (I) converges absolutely, (II) diverges.

15. Evaluate the indefinite integral  $\int \arctan(4x^3) dx$  as a Maclaurin series.

- (a)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+1} x^{6n+4}}{(2n+1)(6n+4)}$
- (b)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+2} x^{6n+6}}{(2n+1)(2n+2)}$
- (c)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+1} x^{6n+6}}{(2n+1)(2n+2)}$
- (d)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n+2} x^{6n+4}}{(2n+1)(6n+4)}$
- (e)  $C + \sum_{n=0}^{\infty} \frac{(-1)^n 4^{6n+4} x^{6n+4}}{(2n+1)(6n+4)}$

Part 2: Work Out

**Directions:** Present your solutions in the space provided. *Show all your work* neatly and concisely and *box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (8 pts) Determine whether the series  $\sum_{n=2}^{\infty} \frac{\sqrt{n} + 3}{n^4 - n}$  converges or diverges. Support your answer.



17. (8 pts) Find the Taylor Series for  $f(x) = \frac{1}{x^3}$  centered at  $x = 2$ .

18. (12 pts) Express  $\int_0^{1/3} \cos(x^3) dx$  as an infinite series.

19. (12 pts) Find (a) the Radius of convergence and (b) Interval of convergence of the power series  $\sum_{n=2}^{\infty} \frac{(-1)^n (4x-1)^n}{9^n (n-1)}$ .