MATH 152, Spring 2022

EXAM III - VERSION B

LAST NAME(print): ______________________________ FIRST NAME(print): ______________________________

UIN: ______________________________

INSTRUCTOR: ______________________________

SECTION NUMBER: ________________

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and ness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam* *on the ScanTron form.***

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____

PART I: Multiple Choice. 4 points each.

1. For which of the following series is the Ratio Test inconclusive?

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n!}$

   (b) $\sum_{n=1}^{\infty} \frac{\cos(n)}{n!}$

   (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4^n}$

   (d) $\sum_{n=1}^{\infty} \frac{n! \sin(n)}{8^n}$

   (e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n}$

2. For what values of p does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converge?

   (a) Only if $p > 1$.

   (b) Only if $p > 0$.

   (c) Only if $p \geq 1$.

   (d) Only if $p \geq 0$.

   (e) The series converges for all p.

3. What is the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-2)^n (2n+1)!}{(n+1)!}$?

   (a) $\{0\}$

   (b) $(-\infty, \infty)$

   (c) $[1.5, 2.5]$

   (d) $\{2\}$

   (e) $[1.5, 2.5)$

4. Given the series $\sum_{n=1}^{\infty} c_n(x+2)^n$ converges at $x=3$. Which of the following statements **must** be true?

(a) The series $\sum_{n=1}^{\infty} c_n(x+2)^n$ diverges at $x=4$.

(b) The series $\sum_{n=1}^{\infty} c_n(x+2)^n$ converges at $x=-5$.

(c) The series $\sum_{n=1}^{\infty} c_n(x+2)^n$ diverges at $x=-8$.

(d) The series $\sum_{n=1}^{\infty} c_n(x+2)^n$ converges at $x=-7$.

(e) None of the above statements must be true.

5. Which of the following statements is true for the three series given below?

(I) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^3}$ (II) $\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$ (III) $\sum_{n=2}^{\infty} \frac{(-1)^n (n)}{\ln n}$

(a) I converges absolutely, II and III converge conditionally.

(b) I and II converge conditionally, and III diverges.

(c) I converges absolutely, II converges conditionally, and III diverges .

(d) I converges conditionally, II and III diverge.

(e) I and II converge absolutely and III converges conditionally.

6. Which of the following is the correct Maclaurin series for $f(x) = x\sin(5x)$?

(a) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1} x^{2n+2}}{(2n+1)!}$

(b) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+2} x^{2n+1}}{(2n+1)!}$

(c) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1} x^{2n+1}}{(2n+1)!}$

(d) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{n} x^{2n+2}}{(2n+1)!}$

(e) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+2} x^{2n+2}}{(2n+1)!}$

7. The series $\sum_{n=1}^{\infty} \frac{\sin(n)+5}{n^{3/2}}$

(a) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{4}{n^{3/2}}$

(b) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{5}{n^{3/2}}$

(c) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{6}{n^{3/2}}$

(d) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(e) None of the above

8. If $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{n+1}}{5^n}(x-2)^n$, find $f^{(25)}(2)$, that is, the 25^{th} derivative of $f(x)$ evaluated at $x = 2$.

(a) $f^{(25)}(2) = \frac{(-1)3^{(26)}(25)!}{5^{(25)}}$

(b) $f^{(25)}(2) = \frac{(-1)3^{(25)}}{5^{(25)}(25)!}$

(c) $f^{(25)}(2) = \frac{(-1)3^{(26)}}{5^{(26)}(25)!}$

(d) $f^{(25)}(2) = \frac{3^{(25)}(25)!}{5^{(26)}}$

(e) $f^{(25)}(2) = \frac{3^{(26)}(25)!}{5^{(25)}}$

9. If we find the third degree Taylor Polynomial for $f(x) = e^{-2x}$ centered at 4, what is the coefficient of $(x-4)^3$?

(a) $-\frac{8}{3}e^{-8}$

(b) $\frac{8}{3}e^{-8}$

(c) $\frac{2}{3}e^{-8}$

(d) $\frac{4}{3}e^{-8}$

(e) $-\frac{4}{3}e^{-8}$

10. Which of the following is a power series representation for $f(x) = \dfrac{1}{(1-3x)^2}$?

(a) $f(x) = \sum_{n=0}^{\infty} 3^n n x^{n-1},\ |x| < \dfrac{1}{3}$

(b) $f(x) = \sum_{n=0}^{\infty} 3^n (n+1) x^n,\ |x| < \dfrac{1}{3}$

(c) $f(x) = -\sum_{n=0}^{\infty} 3^{n-1} (n+1) x^n,\ |x| < \dfrac{1}{3}$

(d) $f(x) = -\sum_{n=0}^{\infty} 3^n n x^{n-1},\ |x| < \dfrac{1}{3}$

(e) $f(x) = \sum_{n=0}^{\infty} 3^{n-1} (n+1) x^n,\ |x| < \dfrac{1}{3}$

11. Find the sum of the series $\sum_{n=0}^{\infty} \dfrac{(-1)^n 3^n (\pi)^{2n}}{n!}$.

(a) 0

(b) -1

(c) $\cos(3\pi^2)$

(d) $e^{3\pi^2}$

(e) $e^{-3\pi^2}$

12. Which of the following is a power series representation for $f(x) = \dfrac{x}{x^3+8}$?

(a) $f(x) = \sum_{n=0}^{\infty} \dfrac{x^{3n+1}}{8^{n+1}},\ |x| < \dfrac{1}{2}$

(b) $f(x) = \sum_{n=0}^{\infty} \dfrac{x^{3n+3}}{8^{n+1}},\ |x| < 2$

(c) $f(x) = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{3n+1}}{8^{n+1}},\ |x| < 2$

(d) $f(x) = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{3n+1}}{8^{n+1}},\ |x| < \dfrac{1}{2}$

(e) $f(x) = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{3n+3}}{8^{n+1}},\ |x| < 2$

13. When we apply the Ratio Test to the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}3^{2n}}{n^2+100}$, we find

(a) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{3}$

(b) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$

(c) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{9}$

(d) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 3$

(e) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 9$

14. Using The Alternating Series Estimation Theorem, what is the minimum number of terms needed to find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ to within $\frac{1}{165}$?

(a) $n = 6$
(b) $n = 3$
(c) $n = 4$
(d) $n = 5$
(e) $n = 7$

True or False. On your scantron, bubble 'a' if true and bubble 'b' if false. One point each.

15. If $0 \le a_n \le b_n$ for every positive integer n and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ converges.

16. If $0 \le a_n \le b_n$ for every positive integer n and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ also converges.

17. If $0 \le a_n \le b_n$ for every positive integer n and $\lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| < 1$, then $\sum_{n=1}^{\infty} a_n$ converges.

18. If $0 \le a_n \le b_n$ for every positive integer n and $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} b_n$ diverges.

PART II: Free Response: Show all work and box your final answer!

19. (12 pts) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+5)^n}{6^n(4n+1)}$. Be sure to test the endpoints of the interval for convergence.

20. Consider $f(x) = x^5 \cos\left(\frac{x}{5}\right)$.

a.) (5 pts) Using the known Maclaurin series for $\cos x$, write $f(x) = x^5 \cos\left(\frac{x}{5}\right)$ as a Maclaurin series. Include the radius of convergence.

b.) (4 pts) Using the result above, evaluate $\int_0^{0.2} x^5 \cos\left(\frac{x}{5}\right)\, dx$ as a series.

21. **Using a comparison test or limit comparison test**, determine whether the series below converge or diverge. Show all work, as illustrated in class, by naming the test, applying the test, and drawing the correct conclusion.

(a) (4 pts) $\sum_{n=1}^{\infty} \frac{6^n}{n+7^n}$

(b) (4 pts) $\sum_{n=3}^{\infty} \frac{1}{\sqrt[5]{2n^5+3n+1}}$

(c) (4 pts) $\sum_{n=1}^{\infty} \frac{\sin n}{n^6+n^4+\sqrt{n}}$

22. (7 pts) Find the Taylor Series for $f(x) = \dfrac{1}{x^3}$ centered at 3. You do not need to find the radius or interval of convergence.

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Question	Points Awarded	Points
1-18		60
19		12
20		9
21		12
22		7
		100

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