MATH 152, Fall 2022

COMMON EXAM II - VERSION A

LAST NAME(print): ______________________ FIRST NAME(print): ______________________

INSTRUCTOR: ______________________

Key

UIN: ______________________

SECTION NUMBER: ____________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam* on the *ScanTron form.***

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________

Some integrals that may or may not be useful.

$$\int \sec^3 x\, dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

$$\int \csc^3 x\, dx = \frac{-1}{2}\csc x \cot x + \frac{1}{2}\ln|\csc x - \cot x| + C$$

PART I: Multiple Choice. 3.5 points each

1. Let $\sum_{n=1}^{\infty} a_n$ be a series whose nth partial sum is $s_n = \frac{7n^2+5}{5n^2+2}$. The series

   (a) converges to $\frac{12}{7}$

   (b) converges to 2.5

   (c) diverges

   (d) converges to $\frac{7}{5}$

   (e) None of these.

$\lim_{n \to \infty} S_n = \frac{7}{5}$

2. After an appropriate substitution, the integral $\int x^2\sqrt{9-x^2}\,dx$ is equivalent to which of the following?

   (a) $9\int \cos^2\theta\, d\theta$

   (b) $81\int \sin^2\theta\cos^2\theta d\theta$

   (c) $27\int \sin^2\theta\cos\theta\, d\theta$

   (d) $81\int \sec^3\theta\tan^2\theta\, d\theta$

   (e) $27\int \sec^2\theta\tan\theta\, d\theta$

$x = 3\sin\theta \qquad dx = 3\cos\theta\, d\theta$

$= \int (3\sin\theta)^2 \sqrt{9 - 9\sin^2\theta} \cdot 3\cos\theta\, d\theta$

$= \int 9\sin^2\theta \sqrt{9\cos^2\theta} \cdot 3\cos\theta\, d\theta$

$= \int 81\sin^2\theta\cos^2\theta\, d\theta$

3. Compute $\int_0^4 \frac{x+2}{x^2+4}\,dx$.

   (a) $\frac{1}{2}(\ln 20 - \ln 4) + \arctan(2)$

   (b) $\ln 6 - \ln 2$

   (c) $\ln 20 - \ln 4$

   (d) $\frac{1}{2}(\ln 20 - \ln 4) + 2\arctan(4)$

   (e) $\ln 20 - \ln 4 + 2\arctan(4)$

$= \int_0^4 \frac{x}{x^2+4}\,dx + \int_0^4 \frac{2}{x^2+4}\,dx$

$= \frac{1}{2}\ln(x^2+4)\Big|_0^4 + 2\cdot\frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big|_0^4$

$= \frac{1}{2}\left(\ln(20) - \ln(4)\right) + \arctan(2) - \arctan(0)$

$= \frac{1}{2}\left(\ln(20) - \ln(4)\right) + \arctan(2)$

4. Let $\sum_{n=1}^{\infty} a_n$ be a series whose nth partial sum is $s_n = \frac{n}{n+2}$. Find a_4.

$$a_4 = S_4 - S_3 = \frac{4}{6} - \frac{3}{5} = \frac{20-18}{30} = \frac{2}{30} = \frac{1}{15}$$

(a) $a_4 = \frac{2}{3}$

(b) None of these.

(c) $a_4 = \frac{1}{21}$

(d) $a_4 = 1$

(e) $a_4 = \frac{1}{15}$

5. Compute $\int_{-1}^{\infty} \frac{1}{1+x^2}\, dx$. $= \lim_{t\to\infty} \int_{-1}^{t} \frac{1}{1+x^2}\, dx = \lim_{t\to\infty} \arctan(x)\Big|_{-1}^{t}$

$$= \lim_{t\to\infty} \left[\arctan(t) - \arctan(-1)\right]$$

$$= \frac{\pi}{2} - -\frac{\pi}{4} = \frac{3\pi}{4}$$

(a) ∞

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) None of these.

(e) $\frac{3\pi}{4}$

6. Which sequence is <u>both</u> bounded and increasing?

(a) $a_n = \sin(2n\pi)$ $\leftarrow$ bounded but not Increasing

(b) None of these.

(c) $a_n = e^{-n}$ $\rightarrow$ decreasing function $f(x) = e^{-x}$

(d) $a_n = 1 - \frac{2}{n}$ $\longrightarrow$ $\lim_{n\to\infty} 1 - \frac{2}{n} = 1$ so this a_n converges. Thus bounded.

(e) $a_n = \ln n$ $\rightarrow$ Increasing but not bounded.

$f(x) = 1 - \frac{2}{x}$

$f' = \frac{2}{x^2} > 0$ for $x > 0$

So Increasing

7. The sequence $a_n = \frac{(-1)^n n^2}{2n^2+5}$ $= (-1)^n b_n$ with $b_n = \frac{n^2}{2n^2+5}$

(a) Diverges

(b) Converges to $\frac{1}{2}$

(c) None of these.

(d) Converges to 0

(e) Converges to $\frac{-1}{2}$

Since $b_n \to \frac{1}{2}$ as $n \to \infty$

a_n will diverge.

8. Which of the following is an appropriate substitution to use when solving the integral $\int \sqrt{16x^2 - 9}\, dx$?

(a) $x = \frac{3}{4}\sin\theta$

(b) $x = \frac{4}{3}\sec\theta$

(c) $x = \frac{4}{3}\sin\theta$

(d) $x = \frac{3}{4}\sec\theta$

(e) $x = \frac{3}{4}\tan\theta$

need $16x^2 = 9\sec^2\theta$

Let $4x = 3\sec\theta$

$x = \frac{3}{4}\sec\theta$

9. Which of the following statements is true regarding the improper integral $\int_1^\infty \frac{1}{e^x + \sqrt{x}}\, dx$?

(a) The integral converges because $\int_1^\infty \frac{1}{e^x+\sqrt{x}}\,dx < \int_1^\infty \frac{1}{\sqrt{x}}\,dx$ and $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ converges.

(b) The integral diverges because $\int_1^\infty \frac{1}{e^x+\sqrt{x}}\,dx > \int_1^\infty \frac{1}{e^x}\,dx$ and $\int_1^\infty \frac{1}{e^x}\,dx$ diverges.

(c) The integral diverges because $\int_1^\infty \frac{1}{e^x+\sqrt{x}}\,dx > \int_1^\infty \frac{1}{\sqrt{x}}\,dx$ and $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ diverges.

(d) The integral converges because $\int_1^\infty \frac{1}{e^x+\sqrt{x}}\,dx < \int_1^\infty \frac{1}{e^x}\,dx$ and $\int_1^\infty \frac{1}{e^x}\,dx$ converges.

(e) The integral converges to 0.

$e^x + \sqrt{x} > e^x$

$\frac{1}{e^x+\sqrt{x}} < \frac{1}{e^x}$

$\int_1^\infty \frac{1}{e^x}\,dx = \lim_{t\to\infty} \int_1^t e^{-x}\,dx = \lim_{t\to\infty} -e^{-x}\Big|_1^t$

$= \lim_{t\to\infty} -e^{-t} - -e^{-1}$

$= 0 + e^{-1} = \frac{1}{e}$

10. Compute the sum of the series $\sum_{n=1}^{\infty} \frac{(-4)^{n+1}}{5^n}$.

(a) This series diverges.

(b) $\frac{16}{9}$

(c) $\frac{-20}{9}$

(d) $\frac{-16}{9}$

(e) None of these.

$\frac{(-4)^2}{5} + \frac{(-4)^3}{5^2} + \frac{(-4)^4}{5^3} + \dots$

$a + ar + ar^2$

So $a = \frac{16}{5}$ $r = \frac{-4}{5}$

Sum $= \frac{a}{1-r} = \frac{\frac{16}{5}}{1-\frac{-4}{5}} = \frac{\frac{16}{5}}{\frac{9}{5}} = \frac{16}{9}$

11. Which of the following is a proper Partial Fraction Decomposition for the rational function

$$\frac{5x+1}{(x+3)(x^2+4x+3)(x^2+4)} \longrightarrow \frac{5x+1}{(x+3)^2(x+1)(x^2+4)}$$

$x^2+4x+3=(x+3)(x+1)$

(a) $\dfrac{A}{x+3}+\dfrac{B}{(x+3)^2}+\dfrac{C}{x+1}+\dfrac{Dx+E}{x^2+4}$

(b) $\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+4x+3}+\dfrac{Dx+E}{x^2+4}$

(c) $\dfrac{A}{x+3}+\dfrac{Bx+C}{(x+3)^2}+\dfrac{D}{x+1}+\dfrac{Ex+F}{x^2+4}$

(d) $\dfrac{A}{x+3}+\dfrac{B}{(x+3)^2}+\dfrac{C}{x+1}+\dfrac{D}{x+2}+\dfrac{D}{x-2}$

(e) None of these.

12. Assume that the sequence $\{a_n\}$ is decreasing and bounded below by 1, i.e. $a_n \geq 1$, for all positive n. Determine if the sequence is convergent or divergent.

$$a_1 = 4 \quad \text{and} \quad a_{n+1} = \frac{10}{7-a_n}$$

as $n \to \infty$ $a_n \to L$

$a_{n+1} \to L$

(a) Convergent to 2

(b) Convergent to 1

(c) Divergent

(d) Convergent to $\dfrac{10}{7}$

(e) Convergent to 5

$L = \dfrac{10}{7-L}$

$7L - L^2 = 10$

$0 = L^2 - 7L + 10$

$(L-5)(L-2)$

$L=5$

$L=2$

13. Which of the following series diverges by the Test for Divergence?

(I) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{\pi n}{2n+1}\right)$ (II) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{4+e^{-3n}}$ (III) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\arctan n}$

(a) (II) only

(b) (III) only

(c) (I), (II), and (III)

(d) (I) and (II) only

(e) (II) and (III) only

I) $\displaystyle\lim_{n\to\infty} \cos\left(\frac{\pi n}{2n+1}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

II) $\displaystyle\lim_{n\to\infty} \frac{4}{4+e^{-3n}} = \frac{4}{4+0} = 1$ II div. by test for div

III $\displaystyle\lim_{n\to\infty} \frac{1}{\arctan(n)} = \frac{1}{\pi/2}$ III div by Test for div.

14. Which of these substitutions would be used to evaluate $\int x^2\sqrt{x^2+4x+13}\,dx$? $= \int x^2 \sqrt{(x+2)^2+9}\,dx$

(a) $x+4=\sqrt{13}\sec\theta$

(b) $x+2=3\tan\theta$

(c) $x^2+4x=\sqrt{13}\tan\theta$

(d) none of these.

(e) $x+2=3\sec\theta$

$x^2+4x+13 = x^2+4x+4+9$

$= (x+2)^2+9$

need $(x+2)^2 = 9\tan^2\theta$

so $x+2 = 3\tan\theta$

15. Let $s=\sum_{n=1}^{\infty}\frac{1}{n^3}$ Using The Remainder Estimate for the Integral Test, determine the smallest value of n that ensures that $R_n = s - s_n \le \frac{1}{44}$.

(a) n = 5

(b) n = 7

(c) n = 8

(d) n = 6

(e) n = 4

need $R_n \le \int_n^{\infty} \frac{1}{x^3}\,dx < \frac{1}{44}$

$\lim_{t\to\infty}\int_n^t x^{-3}\,dx = \lim_{t\to\infty} \left.\frac{x^{-2}}{-2}\right|_n^t = \lim_{t\to\infty} -\frac{1}{2t^2} - -\frac{1}{2n^2}$

$= \frac{1}{2n^2}$ need $\frac{1}{2n^2} < \frac{1}{44}$

$2n^2 > 44$

$n^2 > 22$

so $n=5$

16. The series $\sum_{i=1}^{\infty}\left(e^{1/i}-e^{1/(i+1)}\right)$

(a) converges to e

(b) converges to 0

(c) converges to $e-1$

(d) None of these.

(e) diverges

$S_n = (e^1 - e^{1/2}) + (e^{1/2} - e^{1/3}) + (e^{1/3} - e^{1/4})$

$+ \dots + (e^{1/n-1} - e^{1/n}) + (e^{1/n} - e^{1/n+1})$

$S_n = e^1 - e^{1/n+1}$

$\lim_{n\to\infty} S_n = e^1 - e^0 = e^1 - 1$

$u = \ln(x)$
$du = \frac{1}{x}dx$

$\int \frac{1}{x\ln(x)}dx = \int \frac{1}{u}du = \ln(u) = \ln[\ln(x)]$

17. The improper integral $\int_1^e \frac{1}{x \ln x}\, dx$ $= \lim_{t \to 1^+} \int_t^e \frac{1}{x\ln(x)}dx = \lim_{t\to 1^+} \ln[\ln(x)] \Big|_t^e$

(a) diverges to $-\infty$.

(b) converges to 1.

(c) diverges to ∞.

(d) converges to -1.

(e) converges to $\frac{1}{e} - 1$.

$= \lim_{t\to 1^+} \ln(\ln(e)) - \ln(\ln t)$

$= \infty$

as $t \to 1^+$ $\ln(t) \to 0^+$

thus $\ln(\ln(t)) \to -\infty$

Thus $-\ln(\ln(t)) \to +\infty$

18. The sequence $a_n = \frac{n^2}{n+2} - \frac{n^2}{n+5}$

(a) Converges to 0

(b) None of these.

(c) Converges to 3

(d) Diverges

(e) Converges to 7

$a_n = \frac{n^2(n+5) - n^2(n+2)}{(n+2)(n+5)} = \frac{n^3+5n^2-n^3-2n^2}{n^2+7n+10}$

$= \frac{3n^2}{n^2+7n+10}$

as $n \to \infty$ $a_n \to 3$

PART II WORK OUT

<u>**Directions**</u>: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

19. (6 points) Find a general formula, a_n, for the sequence. Assume the pattern continues, and begins with $n = 1$.

$$\left\{ \frac{-5}{4}, \frac{8}{9}, \frac{-11}{16}, \frac{14}{25}, \frac{-17}{36}, \ldots \right\}$$

Seq. 5, 8, 11, 14, 17

3 3 3

$3n+2$

$a_n = \frac{(-1)^n(3n+2)}{(n+1)^2}$

20. (5 points) Determine whether the series converges or diverges. Fully support your conclusion.

$$\sum_{n=1}^{\infty} \frac{2^{3n}}{7^n} = \frac{2^3}{7^1} + \frac{2^6}{7^2} + \frac{2^9}{7^3} + \dots$$

$$a = \frac{2^3}{7} = \frac{8}{7} \qquad r = \frac{2^3}{7} = \frac{8}{7}$$

Since $|r| = \frac{8}{7} > 1$

The geometric series will diverge.

21. (6 points) Determine whether the series converges or diverges. Fully support your conclusion.

$$\sum_{n=1}^{\infty} ne^{-n^2}$$

$f(x) = xe^{-x^2}$ is continuous, positive, and decreasing on the interval $(1, \infty)$

$$\int_1^\infty xe^{-x^2}\,dx = \lim_{t\to\infty} \int_1^t xe^{-x^2}\,dx$$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$x = 1 \rightarrow u = -1$

$x = t \rightarrow u = -t^2$

$$= \lim_{t\to\infty} \int_{-1}^{-t^2} -\frac{1}{2}e^u\,du = \lim_{t\to\infty} -\frac{1}{2}e^u \Big|_{-1}^{-t^2}$$

$$= \lim_{t\to\infty}\left(-\frac{1}{2}e^{-t^2} - -\frac{1}{2}e^{-1}\right) = \frac{1}{2}e^{-1} = \frac{1}{2e}$$

Since $e^{-t^2} \rightarrow 0$ as $t \rightarrow \infty$

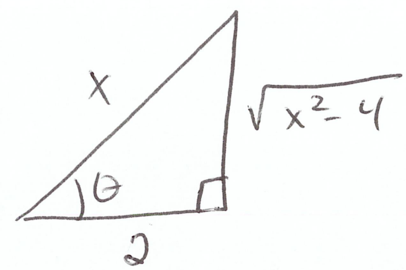
Since $\int_1^\infty xe^{-x^2}\,dx$ converges, the Integral test says that $\sum_{n=1}^{\infty} ne^{-n^2}$ will converge.

22. (10 points) Compute $\int \frac{1}{x^4\sqrt{x^2-4}}\,dx$. In your final answer, any trig or inverse trig expressions that can be rewritten algebraically must be.

let $x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\sec\theta = \frac{x}{2}$



$$\int \frac{2\sec\theta\tan\theta}{(2\sec\theta)^4\sqrt{4\sec^2\theta-4}}\,d\theta = \int \frac{2\sec\theta\tan\theta}{2^4\sec^4\theta\sqrt{4\tan^2\theta}}\,d\theta$$

$$= \int \frac{2\sec\theta\tan\theta}{2^4\sec^4\theta\cdot 2\tan\theta}\,d\theta = \int \frac{1}{2^4}\cdot\frac{1}{\sec^3\theta}\,d\theta = \frac{1}{16}\int \cos^3\theta\,d\theta$$

$$= \frac{1}{16}\int \cos\theta\cos^2\theta\,d\theta = \frac{1}{16}\int \cos\theta\cdot(1-\sin^2\theta)\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \frac{1}{16}\int 1-u^2\,du = \frac{1}{16}\left[u - \frac{u^3}{3}\right] + C$$

$$= \frac{1}{16}\left[\sin\theta - \frac{1}{3}\sin^3\theta\right] + C$$

$$= \frac{1}{16}\left[\frac{\sqrt{x^2-4}}{x} - \frac{1}{3}\left(\frac{\sqrt{x^2-4}}{x}\right)^3\right] + C$$

23. (10 points) Compute $\int \frac{2x^2+5x-5}{(x+1)(x+3)^2}\,dx$

$$\frac{2x^2+5x-5}{(x+1)(x+3)^2} = \frac{A}{x+1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$2x^2+5x-5 = A(x+3)^2 + B(x+1)(x+3) + C(x+1)$$
$$= A(x^2+6x+9) + B(x^2+4x+3) + C(x+1)$$

equating coeff

x^2) $2 = A + B$

x) $5 = 6A + 4B + C$

const) $-5 = 9A + 3B + C$

short cut (evaluate #s)

Let $x=-1$

$$2-5-5 = A(2)^2$$
$$-8 = 4A$$
$$A = -2$$

Since $A+B=2$ we get
$-2+B=2$ or $B=4$

$$5 = 6(-2)+4(4)+C$$
$$5 = -12+16+C$$
$$5 = 4+C \longrightarrow C=1$$

$$\int \frac{-2}{x+1} + \frac{4}{x+3} + \frac{1}{(x+3)^2}\,dx$$

$$= -2\ln|x+1| + 4\ln|x+3| + \frac{-1}{x+3} + C$$

DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-18		63
19		6
20		5
21		6
22		10
23		10
		100