MATH 253.504–506
Exam 1, version A
9/29/99

An unsupported answer is a wrong answer!

1. (10 pts.) Find the equation of the plane which contains the point $(2, 1, -1)$ and is perpendicular to the planes $3x + y - z = 6$ and $2x + 2y + z = 4$.

2. (12 pts.)

   (a) Sketch the domain of $f(x, y) = \sqrt{1 - x^2} + \sqrt{4 - y^2}$.

   (b) What is the range of f? Support your answer!

3. (10 pts.) Does $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2 + xy + y^2}$ exist? Why or why not?

4. (10 pts.) For what value(s) of the constant a, if any, does $u(x, y) = e^{2x}\cos(ay)$ satisfy Laplace's equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$?

5. (10 pts.) Find the equation of the plane tangent to the surface $x^2 + xy - yz^3 + z = 6$ at the point $(2, 1, 1)$.

6. (12 pts.)

   (a) Define: $f(x, y)$ is *differentiable* at a point (a, b).

   (b) Use the definition to prove that $f(x, y) = xy$ is differentiable at every point (a, b).

7. (10 pts.) Find parametric equations of the line tangent to the curve $x = t^2 - 1$, $y = 2t$, $z = t^2 + 1$ at the point $(3, -4, 5)$.

8. (12 pts.) For $f(x, y) = x^2 + 3xy - y^2$,

   (a) What is the directional derivative of f at $(1, 2)$ in the direction from $(1, 2)$ to $(4, 6)$?

   (b) For what unit vector $\vec{u}$ is the direction derivative $D_{\vec{u}} f(1, 2)$ largest?

9. (14 pts.) Suppose that $z = f(x, y)$ where $x = 2u + 3v$ and $y = 3u - 2v$, and that f has continuous second partials.

   (a) Find $\dfrac{\partial z}{\partial u}$ in terms of the partial derivatives of f (in other words, in terms of $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$).

   (b) Find $\dfrac{\partial^2 z}{\partial u^2}$ in terms of the first and/or second partial derivatives of f.