MATH 253.504–506
Exam 3, version A
12/1/99

An unsupported answer is a wrong answer!

This test has 2 pages and 8 problems!

Conversion from spherical to cartesian coordinates:

$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \\ dV &= \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi \end{aligned}$$

1. (15 pts.) Evaluate the line integral $\int_{\mathcal{C}} y\,ds$, where $\mathcal{C}$ is the line segment starting at $(0, 1, 1)$ and ending at $(2, 2, 3)$.

2. (10 pts.) Determine the equation of the plane tangent to the surface $\vec{r}(u, v) = \langle u^2 + v, u^2 - v, uv \rangle$ at the point $(5, 3, -2)$.

3. (15 pts.) For the following, determine if they are vector quantities, scalar quantities, or undefined. Here f is a scalar function and $\vec{F}$ is a vector field.
   (a) $\nabla(\operatorname{div}(\vec{F}))$
   (b) $\operatorname{curl}(\nabla f)$
   (c) $\operatorname{curl}(\vec{F}) + \operatorname{div}(\vec{F})$
   (d) $\operatorname{div}(\nabla f)$
   (e) $\operatorname{curl}(\operatorname{div}(\vec{F}))$

4. (10 pts.) Write a parameterization for the part of the sphere $x^2 + y^2 + z^2 = 4$ which lies outside of the cylinder $x^2 + y^2 = 1$. Be sure to specifiy the parameter domain.

5. (15 pts.) Find $\overline{z}$ of the center of mass of the region above the cone $\phi = \phi_0$ but inside the sphere $\rho = \rho_0$, assuming that density is constant.

6. (15 pts.) Let $\mathcal{R}$ be the region in the x, y plane bounded by $xy = 1$, $xy = 2$, $xy^2 = 1$ and $xy^2 = 3$. Write $\iint_{\mathcal{R}} x\,dA$ as an iterated integral in the variables u and v, where $u = xy$ and $v = xy^2$.

7. (15 pts.) Use Green's theorem to evaluate $\oint_{\mathcal{C}} (x^3 - y^3)\,dx + (x^3 + y^3)\,dy$, where $\mathcal{C}$ is the pictured curve, consisting of two circular arcs and two line segments.

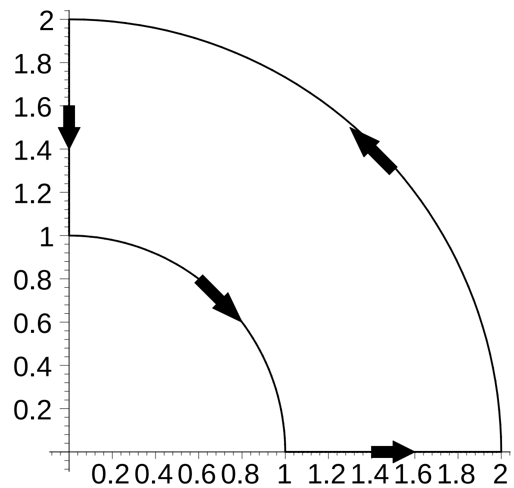

Figure for problem 7.

8. (5 pts.) If $\vec{F}$ is the vector field shown, can $\vec{F}$ be conservative? Why or why not?

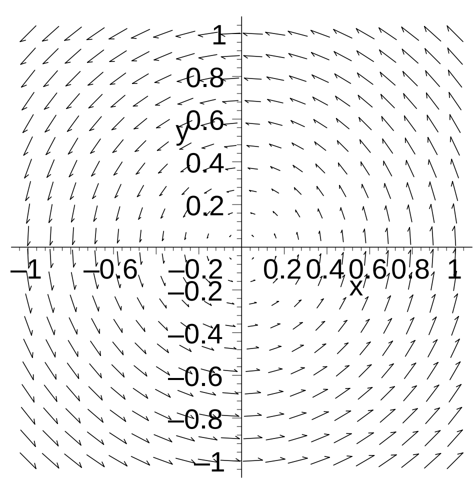

Figure for problem 8.