List of Topics for the Algebra Qualifying Exam

NOTE TO THE STUDENT: *This list of topics, together with the references given below, constitutes a study guide for the Algebra Qualifying Exam. While Math 653 and 654, Algebra I and II, cover much of this material, the exam is separate from the course. Not all of the topics listed below will necessarily be discussed in those courses, and the courses may well cover other material at the discretion of the instructor.*

(1) **Linear and Multilinear Algebra:** The student should be able to do basic computations in real and complex vector spaces and should understand basic notions for vector spaces over a general field. All but the last three topics in this section should be review.
- Vector Spaces
- Subspaces, Quotient Spaces, Products, Duals
- Linear Maps Kernels, Co-Kernels, and Images
- Complexes and Exact Sequences
- Spans, Linear Independence, Basis
- Matrices, Eigenvalues, Eigenvectors, Singular Values
- Inner Products, Symmetric and Orthogonal Matrices
- Canonical Forms
- Quadratic Forms and Bilinear Maps
- Tensor Product

(2) **Group Theory**
- Groups, Subgroups, Normal Subgroups, Factor(Quotient) Groups
- Lagrange's Theorem
- Isomorphism, Homomorphism
- Direct Sum, Direct Product
- Cyclic, Abelian Groups,
- Finitely Generated Abelian Groups
- Permutation Groups
- Cayley's Theorem
- Group Actions
- Sylow Theorems

(3) **Rings and Modules**
- Rings, Fields, Modules
- Homomorphisms
- Exact Sequences of Modules
- Ideals (esp. prime, maximal), Quotient Rings
- Local Rings
- Integral Domains and Fields of Fractions

- Polynomial Rings, PID's, UFD's
- Gauss Lemma and Eisensteins Criterion
- Modules over a PID (Abelian gps, revisited)
- Noetherian Rings and Hilbert Basis Theorem
- Free, Projective and Injective Modules

(4) **Field Theory**
- Extensions
- Finite Fields
- Algebraic Extensions, Separable Extesions, Normal Extensions
- Simple Extensions, Splitting Fields, Galois Extensions
- Cyclotomic Polynomials and Cyclotomic Fields
- Fundamental Theorem of Galois Theory
- Solvablity by Radicals

(5) **References:**
(a) Lang, Algebra, Addison-Wesley
(b) Artin, Algebra, Prentice Hall
(c) Herstein, Topics in Algebra, Ginn and Co.
(d) Hungerford, Algebra, GTM 73, Springer-Verlag