Topics for the Topology / Differential Geometry Qualifying Exam

Topics from items I, II and III a-c are covered in MATH 636, Topology-1, the rest of item III (together with a review of the item III) as well as topics of items IV-VII are covered in MATH 622, Differential Geometry -1.

I. **General topology**
- *a.* Topological spaces; closure and interior of sets. Generating topologies: subspaces, the order topology, the product topology, the quotient topology.
- *b.* Continuous functions and homeomorphisms; the glueing lemma.
- *c.* Connectivity.
- *d.* Compact spaces.
- *e.* Normal spaces, Urysohn's Metrization Theorem; Tietze's extension theorem.
- *f.* The Tychonoff Theorem.

II. **The fundamental group**
- *a.* Homotopy of paths and the fundamental group of the circle.
- *b.* Applications and examples.
- *c.* Seifert-vanKampen theorem.
- *d.* Covering spaces.

III. **Basics of differential manifolds**
- *a.* Differentiable manifolds; examples.
- *b.* Smooth maps, tangent spaces/bundle, differentials of smooth maps.
- *c.* Implicit and Inverse Function Theorems; regular/ critical values/ points. , embeddings, immersions, immersed and embedded submanifolds.
- *d.* Basics on Lie groups: their Lie algebras, General Linear group, Special Linear group, orthogonal group, symplectic group, and their Lie algebras.

IV. **Classical differential geometry of surfaces in $\mathbb{R}^3$**
- *a.* The Gauss map; The first and second fundamental forms; The Gaussian and mean curvatures.
- *b.* The Gauss Egregium theorem. The Gauss-Codazzi equations. The fundamental theorem of surface theory (the Bonnet theorem).
- *c.* The Gaussian curvature of two-dimensional Riemannian manifold.
- *d.* The Gauss-Bonnet theorem.

V. **Basic concepts of tensors and exterior differential calculus**
- *a.* Tensor algebra. Exterior algebra.
- *b.* Vector fields and flows. Lie brackets of vector fields.
- *c.* Tensor fields. The notions of pull-back and push-forward. Lie derivatives of tensor fields.
- *d.* Differential forms and exterior differential of them.

VI. **Stokes' Theorem**

VII. **Distributions, foliations, integral submanifolds, Frobenius theorem.**

References

Topology part:

1. Munkres, James, *Topology*, 2nd edition, xvi+537 pp.
2. Bredon, Glen E., *Topology and geometry.* Graduate Texts in Mathematics, 139. Springer-Verlag, New York, 1997. xiv+557 pp.

Differential Geometry part:

3. Spivak, Michael, *A comprehensive introduction to differential geometry.* Vol. I. Second edition. iv+668 pp.
4. Lee, John M., *Introduction to smooth manifolds.* Second edition. Graduate Texts in Mathematics, 218. Springer, New York, 2013. xvi+708 pp.
5. Nicolaescu, Liviu I., *Lectures on the geometry of manifolds.* xvi+510 pp.
6. Chern, S. S. , Chen, W. H., and Lam, K. S., *Lectures on differential geometry.* x+356 pp.