# Math 689: Nonlinear methods for Learning and Numerical Computation

Instructor: Ronald DeVore

This course is concerned with that part of numerical analysis whose task is to create an approximation $\hat{u}$ to an unknown function $u$ from some given information about $u$. The primary examples are learning a function from (possibly noisy) samples of $u$, computing the solution $u$ to a partial differential equation, or encoding an analog signal. Numerical methods for performing such a task are built on some form of approximation where $u$ is approximated by an element $\hat{u}$ from a set $\Sigma$ of simple functions such as polynomials, splines, wavelets, or neural networks. The older, more traditional methods, chose $\Sigma$ as a finite dimensional linear space of some fixed dimension $n$. The accuracy of the numerical method is improved by increasing $n$.

It is now well understood that the performance of numerical methods can be improved, sometimes dramatically, by replacing linear spaces by nonlinear manifolds $\Sigma_n$ to do the approximation. Examples of such improvements appear in the methods of deep learning and adaptive refinement for solving PDEs. The course will begin by studying various nonlinear methods of approximation and characterizing which functions are well approximated by such methods. The methods to be studied are deep neural networks, adaptive piecewise polynomials, $n$-term wavelet approximation, and greedy algorithms. The course will then turn to targeted applications in learning and numerical PDEs.