MATH 689: MICROLOCAL ANALYSIS

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1. Prerequisites

The main prerequisite for this course is the first year graduate analysis sequence. A familiarity with basic functional analysis, differential geometry, or PDE would be helpful but is not required.

The course should be of interest to students working in any area of analysis, e.g. PDEs, several complex variables, harmonic analysis, numerical analysis, differential geometry, spectral theory, and analytic number theory.

2. Course description

Microlocal analysis consists of various techniques developed (starting in the 1950s and continuing to the present day) to study variable-coefficient and nonlinear partial differential equations. In recent years it has been a valuable tool in many areas related to geometric analysis and spectral theory.

The word "microlocal" in microlocal analysis refers to localization not only in position but in direction as well. Most of the techniques are inspired by the use of the Fourier transform to study constant-coefficient linear partial differential equations.

In this course I will provide a basic introduction to the fundamental tools of microlocal analysis, with a particular emphasis on the notions of pseudodifferential operators (which generalize partial differential operators and provide a convenient way to "almost invert" elliptic operators) and the wave front set (which describes where and in which directions a distribution is not smooth). The applications we emphasize in the course will depend to some extent on the interests of the audience.

3. Course outline

The main topics I intend to introduce in the course are pseudodifferential operators, the notion of the wave front set of a distribution, Hörmander's propagation of singularities theorem, and Fourier integral operators. One of the main applications of the course material is to establish a priori estimates for solutions of PDEs through microlocal techniques. These techniques turn out to be particularly fruitful to the study of elliptic and hyperbolic equations, though they have been adapted in many more contexts as well.

I will of course include most of the necessary "scaffolding" for the course, including the Fourier transform, basic distribution theory, the analytic Fredholm theorem, and any functional analysis necessary to pass from a priori estimates to actual solutions of PDEs.