CD Exam
Texas A&M High School Math Contest
Nov. 8, 2014

Answers should include units when appropriate.

1. A man has \$2.73 in pennies, nickels, dimes, quarters and half dollars. What is the total number of coins he has, if he has equal number of coins of each kind?

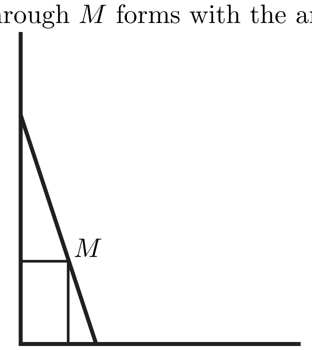
2. One thousand unit cubes are fastened together to form a large cube with edge length 10 units; this is painted and then separated into the original cubes. What is the number of the unit cubes which have at least one face painted?

3. To m ounces of an $m\%$ solution of acid, x ounces of water are added to yield an $(m-10)\%$ solution. Find x.

4. Solve the system

$$\begin{cases} x-y &= 1 \\ x^3-y^3 &= 7. \end{cases}$$

5. Point M is inside a right angle. Distance from M to the sides of the angle are 4 and 8 cm. A line passing through M forms with the angle a right triangle of area 100 cm^2. Find lengths of the legs of the triangle.



6. Solve the system

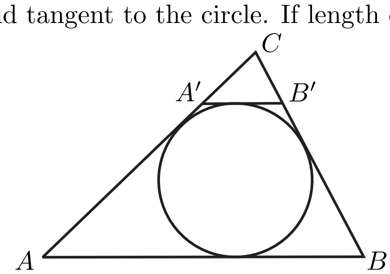
$$\begin{cases} x+y+z &= 6 \\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z} &= 1.5 \\ xyz &= 8. \end{cases}$$

7. In the sequence of numbers $1, 3, 2, \ldots$ each term after the first two is equal to the term preceding it minus the term preceding that. Find the sum of the first one hundred terms of the sequence.

8. Solve the equation

$$(x^2-x-1)^{x^2-1}=1.$$

9. Triangle $\triangle ABC$ has perimeter 20 cm. A circle is inscribed in $\triangle ABC$, and $\overline{A'B'}$ is a line parallel to $\overline{AB}$ and tangent to the circle. If length of $A'B'$ is 2.4 cm, what is the length of AB?



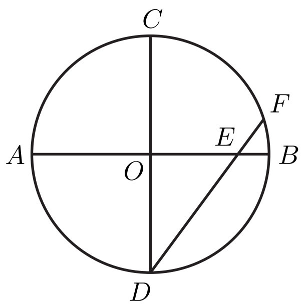
10. Solve the system

$$\begin{cases} 3^{\ln x} &= 4^{\ln y} \\ (4x)^{\ln 4} &= (3y)^{\ln 3}. \end{cases}$$

11. Find all x satisfying the inequality

$$|x-1|+|2-x|>3+x.$$

12. In the figure $\overline{AB}$ and $\overline{CD}$ are diameters of the circle with center O, $\overline{AB}\perp\overline{CD}$, and chord $\overline{DF}$ intersects $\overline{AB}$ at E. If $DE=6$ and $EF=2$, then find the area of the circle.



13. Let $E(n)$ denote the sum of the even digits of n. For example, $E(2014)=2+0+4=6$. Find $E(1)+E(2)+E(3)+\cdots+E(100)$.

14. Find all solutions of the equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$.

15. For which values of a the system

$$\begin{cases} x^2-y^2 &= 0\\ (x-a)^2+y^2 &= 1\end{cases}$$

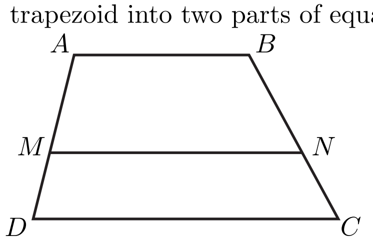
has less than four solutions?

16. Write x^8-16 as a product of quadratic polynomials with real coefficients.

17. Find a positive integral solution of the equation

$$\frac{1+3+5+\cdots+(2n-1)}{2+4+6+\cdots+2n}=\frac{2014}{2015}.$$

18. Let $ABCD$ be a trapezoid, $AB=a$, $CD=b$. Line $\overline{MN}$ is parallel to the bases $\overline{AB}$ and $\overline{CD}$ and cuts the trapezoid into two parts of equal area. Find MN.



19. Let $a_1,a_2,\ldots,a_{2014}$ be the numbers $1,2,\ldots,2014$ written in some order. What is the maximal possible value of $|a_1-a_2|+|a_2-a_3|+|a_3-a_4|+\cdots+|a_{2013}-a_{2014}|+|a_{2014}-a_1|$?

20. How many ordered triples (a,b,c) of integers satisfy

$$|a+b|+c=19 \qquad \text{and} \qquad ab+|c|=27?$$

21. Find all polynomials $P(x)$ such that $xP(x-1)=(x-10)P(x)$ for all x.