EF Exam
Texas A&M Math Contest
08 November, 2014

(NOTE: If units are appropriate, please include them in your answer. All answers must be simplied where possible.)

1. Find the sum of all solutions to the equation $\dfrac{x^2 - 5}{x - 4} = 2x + 9 + \dfrac{11}{x - 4}$.
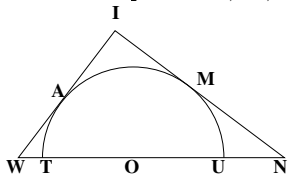
2. If $x < 0$ and $y < 0$, solve the system of equations

$$x^2 + xy + x = 14$$
$$y^2 + xy + y = 28$$

   Write your answer as an ordered pair (x, y).

3. If $\dfrac{1}{\sqrt{x}} + \dfrac{2}{\sqrt{y}} = 2$ and $\dfrac{2}{\sqrt{x}} - \dfrac{1}{\sqrt{y}} = \dfrac{2}{3}$, what is $\sqrt{x} + \sqrt{y}$?

4. In the figure below, semicircle O is tangent to the legs of right triangle WIN at points A and M. Given points T, O, and U lie on $\overline{WN}$, $WT = 3$, and $TU = 24$, what is UN?


5. Circles A, B, and C are tangent to each other and tangent to line ℓ. If the radius of circle A is 18 and the radius of circle B is 8, what is the radius of circle C?
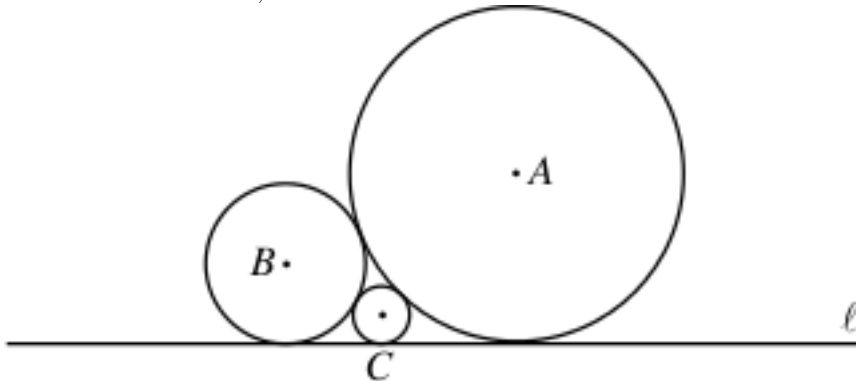
6. A company that rents moving trucks wants to purchase exactly 25 trucks with a combined capacity of 32,000 cubic feet. Trucks are available in small (400 cubic feet capacity), medium (800 cubic feet capacity), and large (1,600 cubic feet capacity). What is the greatest number of large trucks the company can purchase to satisfy their requirements?

7. The expression $\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}$ can be written as $\sqrt{n}$, where n is a positive integer. What is n?

8. Find the product of all solutions in $[0,\,2\pi)$ of the equation $\sin^2 x \sec x + 2\sin^2 x = \sec x + 2$.

9. Find the positive number n such that $n^{\log_{20} 14} = 196$.

10. An eccentric customer came to the bank with 1,000 one-dollar bills and ten bags with the following request: "divide the bills up among the bags so that, if I ask for any dollar amount up to \$1,000, all you have to do is hand me the right bags without having to open any of them". What is the largest amount of money that goes into one bag?

11. Let S be the set of all positive integers with no prime divisors larger than 3 (1, 2, 3, 4, 6, 8, 9, 12, ...). What is the sum of the reciprocals of the elements of S? $\left(1+\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{6}+\dfrac{1}{8}+\dfrac{1}{9}+\dfrac{1}{12}+\cdots\right)$

12. The line $y=-3x+20$ is tangent to the graph of $y=f(x)$ at the point where $x=8$. Evaluate $\displaystyle\lim_{x\to 8}\frac{f(x)+4}{\sqrt[3]{x}-2}$.

13. Find the 2014th derivative of $f(x)=\dfrac{x^{2014}}{x-1}$.

14. Line ℓ is tangent to the curve $x^{2/3}+y^{2/3}=4$ in the first quadrant. What is the length of the line segment of ℓ which lies in the first quadrant?

15. If x, y, and z are positive numbers, find the smallest value of $\dfrac{(x^2+2)(y^2+2)(z^2+2)}{xyz}$.

16. Let A and B be two points on the parabola $y=x^2$, and let C be the point whose tangent line is parallel to $\overline{AB}$. Let Φ be the region between $\overline{AB}$ and the parabola. Find the ratio of the area of Φ to the area of $\triangle ABC$. (See figure below)
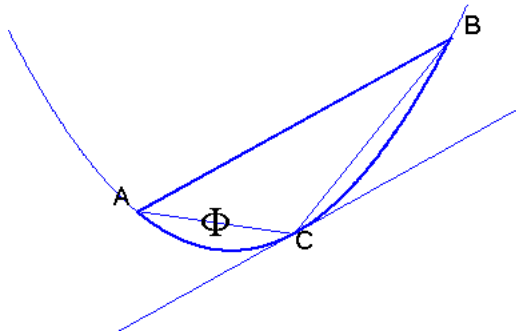
17. Given the points $P(2,\,0)$ and $Q(-1,\,4)$, find the y-coordinate of the point R on the y-axis which maximizes $|PR-QR|$.

18. Given the statement "There exists a straight line which intersects the curve $y=x^4+3x^3+cx^2+2x+4$ in exactly four points." This statement is true if and only if $c\in(-\infty,\,N)$. What is N?

19. Find the minimum value of $|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$ for all x where the expression is defined.