AB Exam
Texas A&M High School Math Contest
November 8, 2014

If units are involved, include them in your answer.

1. What is the largest power of 2 that divides $2^{2013} + 10^{2013}$?

2. It is the year 2014, and Gracie just happens to have 2014 pennies. She uses these pennies to buy Q items, each item costing P pennies. She has 14 pennies left over. How many possibilities are there for the price P?

3. The two roots of the quadratic equation $x^2 - 85x + C = 0$ are prime integers. What is the value of C?

4. Which of the three numbers 2^{100}, 3^{75}, 5^{50} is the largest?

5. How many pairs (x, y) of non-negative integers satisfy $x^4 - y^4 = 16$?

6. The dimensions of a cylinder are changed as follows: the height is increased by $\frac{1}{9}$th of the old height, and the radius is decreased by $\frac{1}{10}$th of the old radius. What is the ratio of the original cylinder's volume to that of the new cylinder?

7. What is the last (units) digit of 2^{2014}?

8. How many polynomials are there of the form $x^3 - 8x^2 + cx + d$ such that c and d are real numbers and the three roots of the polynomial are distinct positive integers?

9. From a group of men and women, 15 women leave. There are then left two men for each woman. From this reduced group 45 men leave. There are then 5 women for each man. How many women were in the original group?

10. Hasse has a 20 gram ring that is 60% gold and 40% silver. He wants to melt it down and add enough gold to make it 80% gold. How many grams of gold should he add?

11. On a test the passing students had an average of 83 while the failing students had an average of 55. If the overall class average was 76, what percent of the class passed?

12. Find all solutions to $|5x - 2| + |5x + 1| = 3$.

13. If $x^2 - 2x - 3$ is a factor of $x^4 + px^2 + q$, what is the value of p?

14. The parabola $y = ax^2 + bx + 1$ has a maximum at $(2, 2)$. What is the value of b?

15. For all real numbers x and y that satisfy $(x + 5)^2 + (y - 12)^2 = 14^2$, find the minimum value of $x^2 + y^2$.

16. The xy-plane is divided into four quadrants: I, II, III and IV. If the point (x, y) satisfies $2x + 3 < y < -\frac{x}{2} - 5$, in what quadrant(s) could (x, y) be?

17. What is the largest integer n such that $\frac{n^2-38}{n+1}$ is an integer?

18. The six digit number $3730A5$, where A is the tens digit, is divisible by 21. Find all possible values of A.

19. How many pairs of integers (x, y) satisfy the equation $y = \frac{x+12}{2x-1}$?

20. Among all of the points (x, y) on the line $2x + 3y = 6$, find the value of x that gives the smallest value of $\sqrt{x^2 + y^2}$.

21. All of the positive integers are written in a triangular pattern beginning as follows and continuing in the same way:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & 3 & 4 & & \\ & 5 & 6 & 7 & 8 & 9 & \\ 10 & 11 & 12 & 13 & 14 & 15 & 16 \end{array}$$

Which number appears directly below 2014?

22. Daisy has twenty 3¢ stamps and twenty 5¢ stamps. Using one or more of these stamps, how many different amounts of postage can she make?

23. How many triples (x, y, z) of <u>rational</u> numbers satisfy the following system of equations?

$$x + y + z = 0$$
$$xyz + z = 0$$
$$xy + yz + xz + y = 0.$$