If units are involved, include them in your answer, which needs to be simplified.

1. If an arc of 60° on circle I has the same length as an arc of 45° on circle II, what is the ratio of the area of circle I to the area of circle II?

2. How many positive integers are there, which are less than 1000 and not divisible by 5 or 7?

3. In a circle of radius r centered at O a chord AB of length r is drawn. From O a perpendicular to AB meets AB at M. From M a perpendicular to OA meets OA at D. In terms of r, what is the area of triangle MDA?
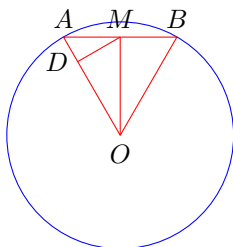
4. The figure below is a half circle with $AD = 3$ inches and $DC = 12$ inches. Line BD is perpendicular to the half circles diameter AC and intersects the circle at point B. How long is line BD?
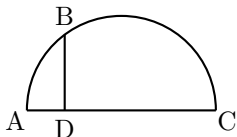

5. The first 50 terms of an arithmetic series sum to 200 and the next 50 terms sum to 2700. What is the value of the first term?

6. The figure below shows two intersecting circles. The circle to the left is centered at the origin and has radius 1. The second circle is centered at $(R, 0)$, and it intersects the unit circle orthogonally, i.e., perpendicularly. The second circle also passes through the point (r, h), which is interior to the unit circle. Find R in terms of r and h.
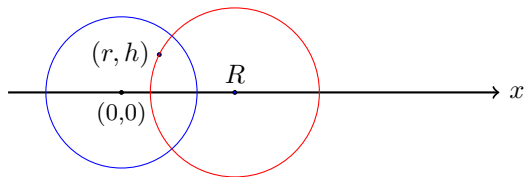

7. Suppose the polynomial $p(x) = x^{2014} - c_1 x^{2013} + c_2 x^{2012} + \cdots + c_{2014}$ has roots $\{\pm 1, \pm 2, \cdots, \pm 1007\}$. What do c_1 and c_{2014} equal?

8. Triangle ABC is isosceles with base AC. Points P and Q are respectively in CB and AB such that $AC = AP = PQ = QB$. What are the number of degrees in angle B?

9. Thirty one books are arranged from left to right in increasing order of price, and the price of each book differs by $2.00 from each adjacent book. Moreover, the price of the most expensive book equals the sum of the prices of the middle book and a book adjacent to the middle book. What is the price of the most expensive book?

10. The function $p(x) = ax^2 + bx + c$ describes a parabola. Suppose the parabola's vertex is located at the point $(-1, 2)$. What does b/a equal?

11. Jane can mow a field in 12 hours, while Jane and Bill working together can mow the field in 8 hours. How long will it take Bill to mow the field by himself?

12. The probability that event A occurs is $7/8$, and the probability that event B occurs is $5/6$. What is the smallest possible value of the probability of $A \cap B$?

13. Triangle ABC in the figure below has area 10. Points D, E, and F all distinct from the vertices of the triangle lie on sides AB, BC, and CA respectively; AD=2 and DB =3. If triangle ABE and quadrilateral DBEF have equal areas, what is that area?
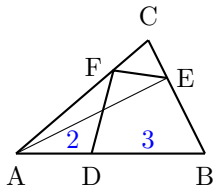
14. In $\triangle ABC$, a point D in on AC so that $AB = AD$. If $\angle ABC - \angle ACB = \pi/6$, what does $\angle CBD$ equal?
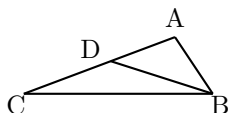

15. Daisy has twenty 3¢ stamps and twenty 5¢ stamps. Using one or more of these stamps, how many different amounts of postage can she make?

16. In the figure below
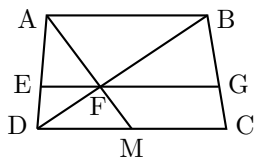
ABCD is a trapezoid with AB and DC parallel. AM is a median of $\triangle ADC$, DB is a diagonal of the trapezoid with the median AM meeting the diagonal DB at F. Line EG passes through F and is parallel to DC. If $\triangle AEF$ has area equal to 6 sq. cm., what is the area of $\triangle BFG$?

17. Triangle ABC has a right angle at C, $AC = 2$, and $BC = 3$. The bisector of $\angle BAC$ meets BC at D. Find CD.
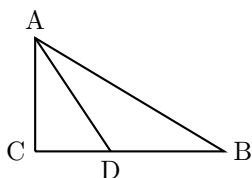
18. A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. Find the ratio of the number of cubes with exactly two green faces to the number of cubes with exactly three green faces.

19. The circle shown below is centered at O, has radius equal to 1, and $\theta = 24°$. What is the sum of angles ACB and OAB?