BC Exam
Texas A&M High School Math Contest
October 22, 2016

All answers must be simplified, and if units are involved, be sure to include them.

1. Given
$$A = \frac{1}{2-\sqrt{3}}$$
and
$$B = (\sqrt{5} - \sqrt{2}\sqrt{\sqrt{3}})(\sqrt{5} + \sqrt{2}\sqrt{\sqrt{3}}),$$
find $2A + B$ simplifying as much as possible.

2. Let x and y be the solutions of the system of equations
$$\sqrt{39 - 2x - 10y} = 5$$
and
$$\sqrt{15 - 2x + 2y} = 5.$$
Find $x + y$.

3. A collection of nickels and dimes has a total value of \$2.40 and contains 35 coins. How many nickels are in the collection?

4. A box containing 180 cubic inches is constructed by cutting from each corner of a cardboard square a small square with side 5 inches, and then turning up the sides. Find the area of the original piece of cardboard.

5. Find the largest common divisor for the numbers
$$11^{100} + 11^{101} + 11^{102} + 11^{103}$$
and
$$7^{100} + 7^{101} + 7^{102} + 7^{103}.$$

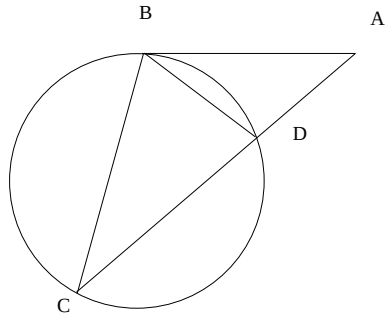
6. Find the sum of all solutions of the equation
$$x^2 + 6x + \sqrt{x^2 + 6x} = 20.$$

7. Suppose that $\triangle ABC$ is a right triangle with $\angle A = 90^\circ$, $AB = 5$, and $AC = 12$. On the line AB we consider the point M such that $\triangle BMC$ is isosceles with $BM = CM$. Find AM.

8. Mr. Kaye is 11 times as old as his daughter Lynn. Thirty-six years from now he will be at most twice as old as Lynn. At most, how old is Lynn?

9. In the $\triangle ABC$, $AB = AC$ and $\angle A = 120^\circ$. The median AD to the side BC is extended through the point D with the segment $DM = 3AD$. Find $\angle DMB$.

10. In the figure below AB is tangent to the circle. If $AB = 8$ and AC exceeds AD by 12, what is AC?



11. Find the sum of all positive integers x for which $x + 56$ and $x + 113$ are perfect squares.

12. Consider the sum
$$S = \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}+\sqrt{n+1}},$$
where n is a positive integer. If $S = 10$, what is the value of n?

13. Find the product of all solutions of the equation
$$(3x^2 - 4x + 1)^3 + (x^2 + 4x - 5)^3 = 64(x^2 - 1)^3.$$

14. A circle whose center is on the x-axis passes through the points $(3, 5)$ and $(6, 4)$. Find the radius of the circle.

15. Find the sum of all integers N with the property that $N^2 - 71$ is divisible by $7N + 55$.

16. In the $\triangle ABC$, BD is the median to the side AC, DG is parallel to the base BC (G is the point of intersection of the parallel with AB). In the $\triangle ABD$, AE is the median to the side BD and F is the intersection point of DG and AE. Find $\dfrac{BC}{FG}$.

17. The function
$$f(x) = x^2 + (x+2)^2 + \cdots (x+98)^2 - \left[(x+1)^2 + (x+3)^2 + \cdots (x+99)^2\right]$$
is a linear function, $f(x) = ax + b$. Find $a - b$.

18. In the isosceles $\triangle ABC$ with $AB = AC$, let AM be the median to the side BC and let BD be the altitude to the side AC. If $\angle AMD = 4\angle BDM$ find $\angle ACB$.

19. Let f and g be two linear functions such that
$$f(x-1) = 2x - 3 + g(1) - f(1)$$
and
$$g(x-1) = 4x + 5 - g(1) - f(1),$$
for all real numbers x. Find $g(5)$.

20. Consider $\triangle ABC$ with $\angle B = \angle C = 70^\circ$. On the sides AB and AC we take the points F and E, respectively, so that $\angle ABE = 15^\circ$ and $\angle ACF = 30^\circ$. Find $\angle AEF$.