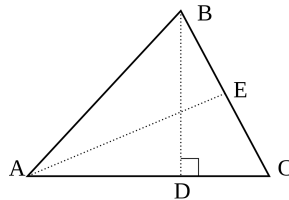


Math Contest, Fall 2017  
BC EXAM

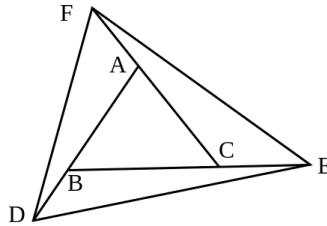
1. List  $x, y, z$  in order from smallest to largest fraction:

$$x = \frac{111110}{111111}, y = \frac{222221}{222223}, z = \frac{333331}{333334}$$

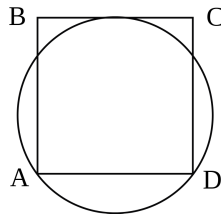
2. A vase in a flower shop contains ten identical red and four identical pink roses. If you would like to pick one red and two pink ones, in how many ways can you do this?
3. Three-digit number  $AB8$  is 296 larger than two-digit number  $AB$ . What is the value of the two-digit number  $AB$ ?
4. The perimeter of an equilateral triangle is equal to the perimeter of a regular hexagon. What is the ratio of their areas (triangle area to hexagon area)?
5. When a number is divided by 5, the remainder is 2. What is the remainder when the number is multiplied by 8 and then divided by 5?
6. Jane has 6 sons and no daughters. Some of her sons have 6 sons each, and the rest have none. Jane has a total of 30 sons and grandsons, and no great-grandsons. How many of Jane's sons and grandsons have no sons?
7. Find all natural numbers  $n$  for which  $2^n + 1$  is divisible by 3.
8. If John walks to his job from home and takes a bus on the way back, he spends 1.5 hour on round trip. If he takes a bus both ways, the round trip takes 30 min. How long (in hours) will it take him to walk both ways?
9. Find all possible values of an integer  $N$  such that  $N^2 - 71$  is divisible by  $7N + 55$ .
10. In an arithmetic sequence there are 10 numbers. The sum of terms at even places is 50, and the sum of terms at odd places is 35. Find the first term,  $a_1$ , and the difference,  $d$ , of this sequence.
11. In a math class of 50 students, the average score on the final exam is 68. The best ten exams are all 100. Find the average of the other 40 exams.
12. If both roots of the quadratic equation  $x^2 - 85x + c = 0$  are prime numbers, what is the value of  $c$ ?
13. In an acute triangle  $ABC$ ,  $BD$  is the altitude and  $AE$  is the median. Given that the measure of the angle  $\angle ECA$  is twice the measure of the angle  $\angle EAC$  and that  $BC = 10$  cm, find  $AD$ .



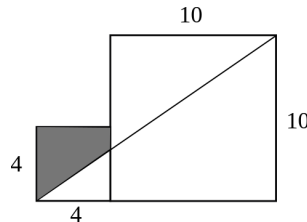
14. It is known that for some  $x$  the value of the expression  $((x + 2x) \cdot 3x - 4x) \div 5x$  stays the same even when all parenthesis are removed. What is the value of  $x$ ?
15. How many positive integers  $n$  have the property that when 1,000,063 is divided by  $n$ , the remainder is 63?
16. Determine all integer values of the parameter  $a$  for which the equation  $\frac{3}{a-2x} = \frac{5}{ax-4}$  has a negative solution.
17. Each side of a triangle  $\triangle ABC$  is extended as shown so that  $BD = \frac{1}{2}AB$ ,  $CE = \frac{1}{2}BC$ , and  $AF = \frac{1}{2}CA$ . What is the ratio of the area of the triangle  $\triangle DEF$  to the area of the triangle  $\triangle ABC$ ?



18. Given a square  $ABCD$  with side length of 8 inches, a circle is drawn through vertices  $A$  and  $D$  and tangent to side  $BC$ . What is the radius of the circle in inches?



19. Consider two adjacent squares with sides 4 and 10 meters. Find the area of the shaded region.



20. A circle is inscribed into a right triangle. From the center of the circle a perpendicular is dropped on the hypotenuse, dividing the hypotenuse into two segments of length 12 and 5. Find the legs of the triangle.
21. Consider a square  $ABCD$  and choose points  $E$  and  $F$  on the square such that  $\triangle BEF$  is equilateral. What is the ratio of the area of  $\triangle DEF$  to that of  $\triangle ABE$ ?
22. Given a sequence of integers  $\{a_n\}$  such that  $a_1 = 1$  and  $a_{m+n} = a_m + a_n + mn$  for  $m, n = 1, 2, 3, \dots$  find  $a_{15}$ .