

## 2018 AB Exam

Texas A&M High School Math Contest

October 20

*All answers must be simplified, and if units are involved, be sure to include them.*

1. Find the rational number in the open interval  $(\frac{1}{3}, \frac{1}{2})$  that has the smallest positive denominator.
2. Let  $L$  be the line with equation  $ax + by = c$  where  $abc \neq 0$ . Let  $M$  be the reflection of  $L$  across the  $y$ -axis. Let  $N$  be the reflection of  $L$  across the  $x$ -axis. Which of the following must be true about  $M$  and  $N$ ?
  - (A) The  $x$ -intercepts are equal.
  - (B) The  $y$ -intercepts are equal.
  - (C) The slopes are equal.
  - (D) The slopes are reciprocals.
  - (E) The slopes are negative reciprocals.
3. The sum of two natural numbers  $a$  and  $b$  is equal to 153. What is the largest possible value of their greatest common divisor,  $\gcd(a, b)$ ?
4. Find all ordered pairs  $(a, b)$  such that  $a + b = 16$  and  $\frac{1}{a} + \frac{1}{b} = \frac{4}{7}$ .
5. A certain medication has an 80% success rate of curing people who have a specific illness. If three people with the illness are selected at random and take the medicine, what is the probability that exactly two of the three people will be cured?
6. If  $ax + 3y = 5$  and  $2x + by = 3$  represent the same line, find the value of  $a + b$ .
7. Let  $S = \{1, 2, 3, \dots, 11\}$ . A subset of  $S$  of size 3 is said to be “special” if it contains at most one odd integer. If a subset of  $S$  of size 3 is chosen at random, what is the probability it is special?
8. When the integer  $D > 1$  is divided into each of the numbers 1059, 1417, 2312 the same remainder  $R$  is obtained. Find  $D$ .
9. Determine all linear functions  $f(x) = ax + b$  such that  $f(x) - f^{-1}(x) = 44$  for all  $x$ . Note: the symbol  $f^{-1}$  denotes the inverse function of  $f$ , and not its reciprocal.
10. For some real values of  $p$ , two of the roots of  $x^3 + px^2 + 12x - 9 = 0$  have a sum of 4. Find the third root.
11. The equation  $(x^2 - x + 1)(x^2 - x + 2) = 12$  has two real solutions. Find their product.
12. Hasse traveled 1 hour longer and 2 miles farther than Daisy but averaged 3 mph slower. If the sum of their times was 4 hours, what was the sum in miles of the distances they traveled?
13. Two numbers when written in base  $a$  are 32 and 24. These same two numbers written in base  $b$  give 43 and 33 respectively. Find the sum of the two numbers in base 10.

14. The average age of the people in a group of men and women is 31 years. The average age of the men is 35 and the average age of the women is 25. What is the ratio of the number of men to the number of women? Write your answer as a simple fraction.
15. The point  $(1, 2)$  is on the line  $y = 2x$ . Find the  $x$ -coordinate of each of the two points on  $y = 2x$  that are 10 units from  $(1, 2)$ .
16. For exactly two real values  $m_1$ , and  $m_2$  of  $m$  the line  $y = mx + 3$  intersects the parabola  $y = x^2 + 2x + 7$  at exactly one point. Find  $m_1 + m_2$ .
17. Suppose  $f$  and  $g$  are functions such that  $f(x) = 2x + 1$  and  $g(f(x)) = 4x^2 + 1$ , find  $g(x)$ .
18. A stock loses 10% of its value on Monday. On Tuesday it loses 20% of the value it had at the end of the day on Monday. What is the overall percent loss in value from the beginning of Monday to the end of Tuesday?
19. Solve for  $x$  the following equation

$$\sqrt{x} + \sqrt{x - 1} = 2.$$

20. Daisy bought a mixture of two types of her favorite candies costing 40 cents and 50 cents per ounce respectively. The cost of 50 ounces of her mixture was \$21.80. How many ounces of the 50-cent candy were in the mixture?
21. One gear turns  $33\frac{1}{3}$  times a minute. Another gear turns 45 times a minute. Initially a mark on each gear is pointing due north. After how many seconds will the two gears next have both marks pointing due north?