2019 AB Exam

Texas A&M High School Math Contest

November 9, 2019

All answers must be simplified, and if units are involved, be sure to include them.

1. What is the equation of the axis of symmetry for the graph of $x = y^2 + 8y + 30$?

2. A positive integer n written in base b is 25_b. If $2n$ is 52_b, what is b?

3. Let n be a positive integer and let $S = 1 + 2 + 3 + \cdots + 10^n$. How many factors of 2 are there in the prime factorization of S?

4. (Multiple choice) In the real number system the polynomial $x^4 + 4$

   (A) does not factor into two polynomials of smaller degree.

   (B) factors in a cubic and a linear polynomial.

   (C) factors into two quadratics.

   (D) factors into four linear polynomials.

   (E) factors into a quadratic and two linear polynomials.

5. On a math exam every question gives two explicit choices, A and B, but the answer can be any combination of these two choices (*A and not B*, *not A and not B*, etc.) The answer includes choice A 70% of the time and choice B 50% of the time. The answer *neither A nor B* occurs 10% of the time. For what percentage of questions is the answer *both A and B*?

6. The numerals for the counting numbers are written in (increasing) order beginning with 1. What is the 125th digit written?

7. The correct formula for converting a Celsius temperature (C) to a Fahrenheit temperature (F) is given by $F = \frac{9}{5}C + 32$. To approximate the Fahrenheit temperature, Hasse doubles C and then adds 30. What is the largest magnitude (absolute value) of the error in this approximation for $-20 \leq C \leq 35$?

8. Find all values of x such that
$$\frac{x^2 + x + 4}{2x + 1} = \frac{4}{x}.$$

9. Given a quadratic polynomial $p(x) = x^2 + bx + c$ such that $p(2) = 0$. Find the value of $p(-1) + p(5)$.

10. A bag contains 40 balls each of which is either black or gold. Mike reaches into the bag and randomly removes two balls. Each ball in the bag is equally likely to be removed. If the probability that two gold balls are removed is $\frac{5}{12}$, how many of the 40 balls are gold?

11. A 200 gram solution consists of water and salt. 25% of the total mass of the solution is salt. How many grams of water needs to be added in order to change the solution so that it is 10% salt by mass?

12. What is the largest two-digit integer that becomes 75% greater when its digits are reversed?

13. The equation $4x^2 - y^2 = 11$ has exactly one integer pair solution (x, y) with both $x > 0$ and $y > 0$. Find it.

14. Compute the area of the region bounded by the graphs of $y = 4 - |x|$ and $y = |x| - 4$.

15. Determine all ordered pairs (a, b) of real numbers that satisfy the following system of equations

$$a + b = 16,$$

$$\frac{1}{a} + \frac{1}{b} = \frac{4}{7}.$$

16. The numbers a, b, c are in geometric progression in that order, and they are the lengths of the sides of a right triangle with c being the length of the hypotenuse. Find $\frac{b}{a}$.

17. If $\frac{a}{b}$ is the <u>reduced</u> fraction that equals $0.571717\ldots$, find $a + b$.

18. The parabola $y = x^2 - 4x + 7$ is reflected about the line $y = 3$ to obtain another parabola. Find an equation for this parabola in the form $y = ax^2 + bx + c$.

19. Daisy likes to paddle her raft down a river from point A to point B. The speed of the current in the river is always the same. Daisy always paddles the raft at the same constant speed. On days when she paddles with the current, it takes her 18 minutes to go from A to B. When she does not paddle the current carries her from A to B in 30 minutes. If there were no current, how long would it take her to paddle from A to B?

20. Suppose L is a line that passes through the first quadrant. The positive x-axis, the positive y-axis and L form the boundary of a triangular shaped region whose area is 20. If L contains the point $(6, -5)$, find its y-intercept.