

AB Exam
Texas A&M Math Contest
4 November, 2023

(NOTE: If units are appropriate, please include them in your answer. All answers must be simplified where possible.)

1. Find the exact value of $\frac{77.85 \times \frac{3217}{100} - \frac{7785}{100} \times 11.94}{20.23 \times \frac{832}{1000} + \frac{668}{1000} \times 20.23}$.
2. There are 120 different 5-digit numbers that use each of the digits 2, 4, 5, 7, and 8 exactly once. If listed in numerical order from smallest to largest, what number is in the 97th position of the list?
3. A resale store sells certain items at a price 50% below the original price. On Saturdays, these items are discounted an additional 20% off the sale price. If you purchase one of these items on a Saturday, what percentage of the original price do you pay?
4. Your friend has a list of seven numbers. They tell you (truthfully) that the average of the first four numbers is 5, the average of the last four numbers is 8, and the average of all seven numbers is $6\frac{4}{7}$. What is the middle number?
5. Write $\left(1 - \frac{2}{3}\right) \left(1 - \frac{2}{4}\right) \left(1 - \frac{2}{5}\right) \cdots \left(1 - \frac{2}{2022}\right) \left(1 - \frac{2}{2023}\right)$ as a single fraction.
6. BT Parnum has a traveling Brass Band & Rhino Circus. The group involved had a total of 150 legs and 150 horns. If each band member had two legs and packed three horns, and each rhinoceros had four legs and the usual one horn, what is $\frac{R}{B}$, the ratio of the number of rhinoceroses to the number of band members.
7. What is the remainder when $(2023)^1 + (2023)^2 + (2023)^3 + \cdots + (2023)^{2022} + (2023)^{2023}$ is divided by 10?
8. If p and q are the roots of $x^2 + 2023x + 114$, what is $\frac{1}{p} + \frac{1}{q}$?
9. A (really big) gumball machine contains 2023 blue gumballs, 2023 red gumballs, 2023 green gumballs, and 2023 yellow gumballs. If each gumball costs 1 cent, what is the least amount of money you have to spend in order to guarantee you'll have at least 4 gumballs of the same color (any color)?

10. The sum of two natural numbers a and b is 2023. What is the largest possible value of their greatest common divisor, $\gcd(a, b)$?
11. Let x be a solution to the equation $\sqrt{3-x} = 1-x$. Find the sum of all possible values of x .
12. Find the smallest value of m for which the equation $x^2 + 2mx + 3m^2 + m - 21 = 0$ has real solutions.
13. The product of three consecutive integers is divided by the first integer. Then the product of the three is divided by the second integer. Finally, the product of the three is divided by the third integer. If the sum of the three quotients is 254 more than three times the square of the smallest integer, what is the smallest integer?
14. You live in a state whose income tax is $p\%$ of the first \$28,000, then $(p+2)\%$ of any amount above \$28,000. Last year, your overall income tax amounted to $(p+0.25)\%$ of your income. What was your income last year?
15. How many positive integers from 1 to 2023 are multiples of 3 or 4, but not multiples of 5?
16. The sum of the reciprocals of two numbers x and y is 4, and the sum of the squares of the reciprocals is 11. What is the product of the two numbers?
17. A curve consists of all the points (x, y) which are vertices of a parabola of the form $y = x^2 + 2ax + a$ for some real number a . Find the equation of the curve.
18. How many pairs of integers (x, y) are solutions to the equation $\sqrt{x} + \sqrt{y} = \sqrt{5290}$?
19. You start with a positive integer N and create a sequence of numbers, where the next number is obtained by subtracting the largest possible perfect square from the current number until 0 is obtained. For example, if you start with 23, your sequence is 23, 7 (23-16), 3 (7-4), 2 (3-1), 1 (2-1), and 0 (1-1), which contains 6 numbers. You manage to find a sequence which contains 2023 numbers. What is the ones digit of the smallest possible starting number N ?
20. Four sparrows found a dish of seeds
 Fine birdie food, was free of weeds
 Said Pip: "In turn each take two grains,
 Plus a third of what remains.
 I'll go first, then Pep, then Pop,
 With Pap the last. And then we stop."
 But Pap cried out, "It isn't fair.
 Mine's two seeds less than half Pep's share."
 But Pip was boss; his word was law
 So little Pap got nothing more.
 Poor Pap, his share was rather small!
 How many seeds were there in all?