Eradicating Invasive Species through Sex Reversal

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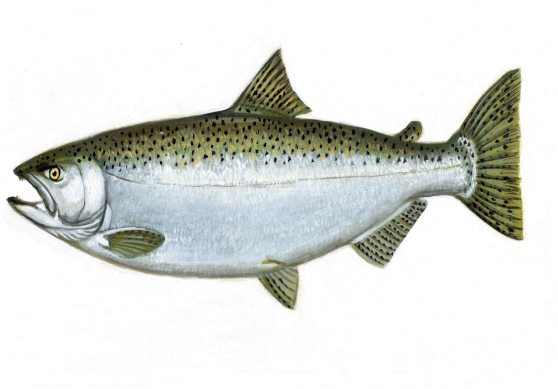
Sex Reversal Background

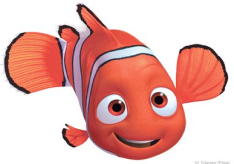
- Chinook Salmon studied in the Columbia River, 2001
- Females discovered with Y chromosomes
- Hormonal sex reversal leads to long-term reduction in XX Females



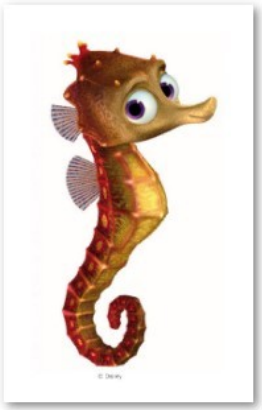
The Trojan Y Chromosome Model

- Invasive species eradication

- Start with a species of female XX fish and male XY fish

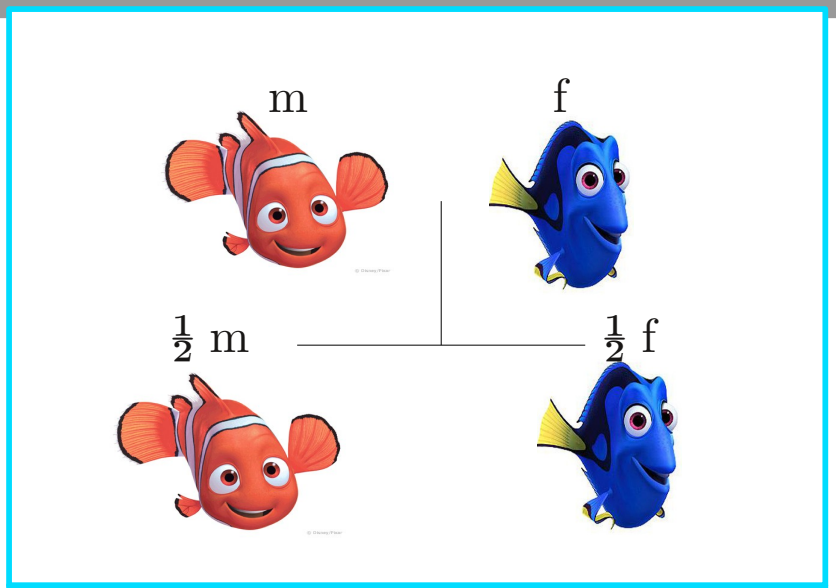


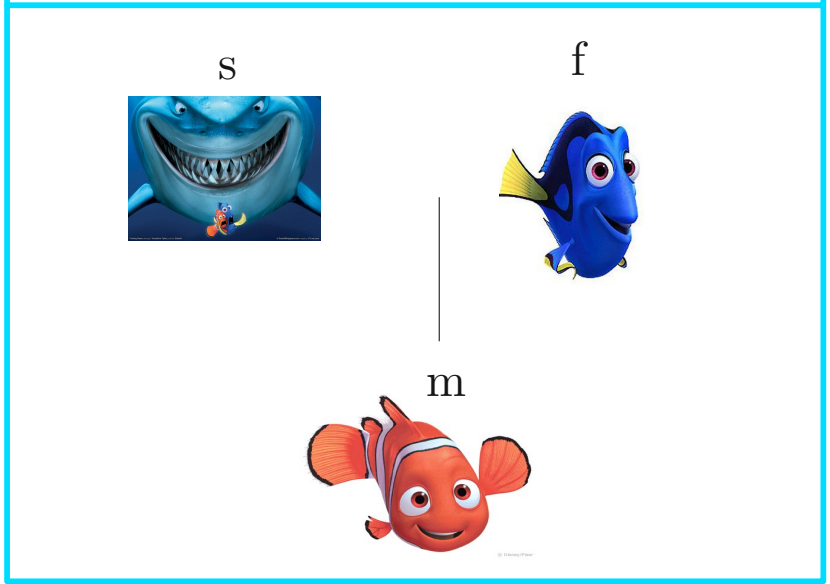
- YY supermale fish cultivated in aquaculture

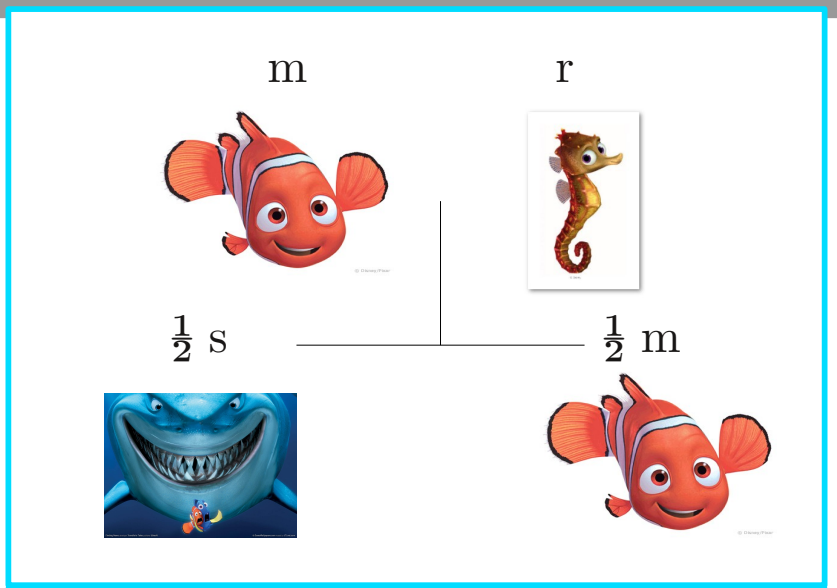


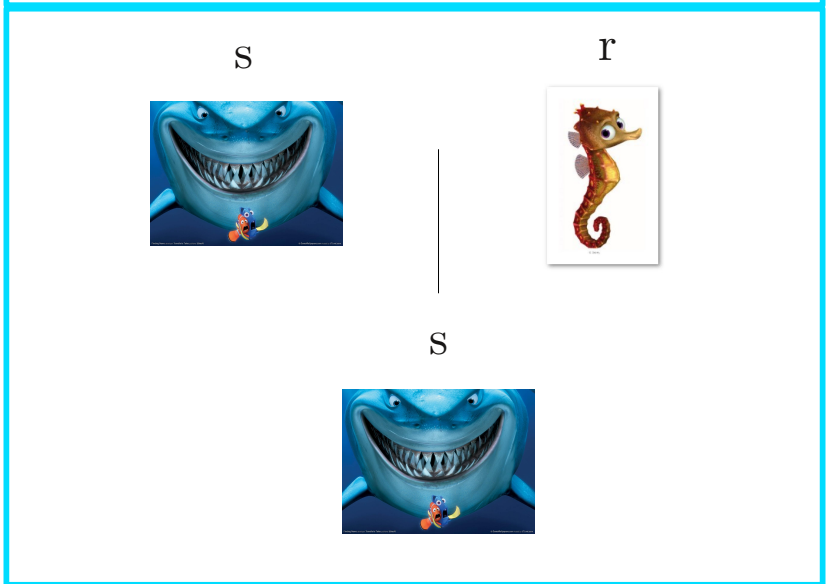
- Feminized YY supermales added to the wild population

Reproductive Options









ODE Model

$$\frac{df}{dt} = 1/2\, fm\, \mathrm{B}\, L - \delta\, f$$

$$\frac{ds}{dt} = (1/2\, r\, m + r\, s)\, \mathrm{B}\, L - \delta\, s$$

$$\frac{dm}{dt} = (1/2\, fm + 1/2\, r\, m + f\, s)\, \mathrm{B}\, L - \delta\, m$$

$$\frac{dr}{dt} = \mu - \delta\, r$$

B = Birth coefficient

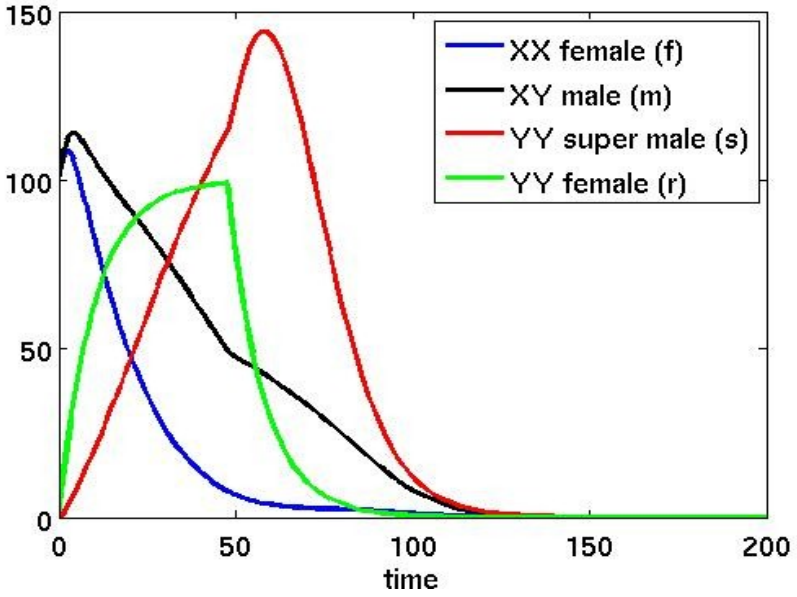
δ = Death coefficient

μ = rate of introduction of feminized supermales

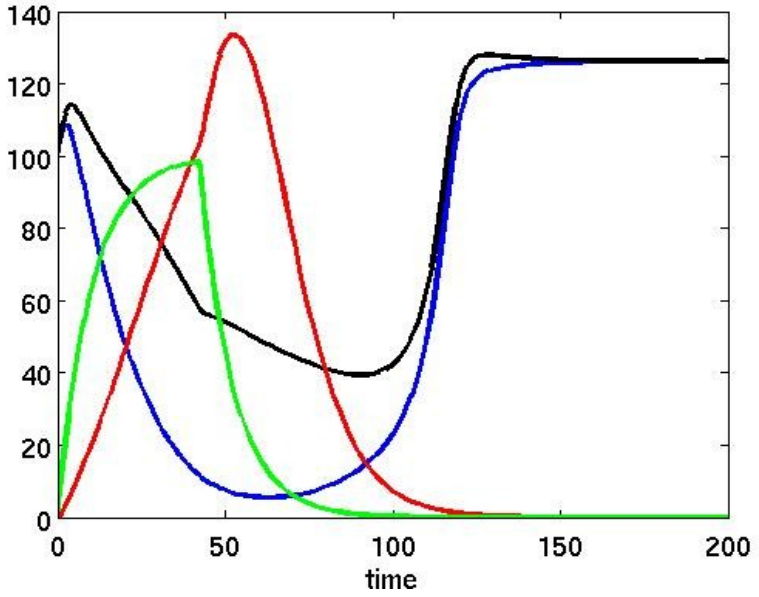
K = carrying capacity $\qquad L = 1 - \dfrac{f + m + r + s}{K}$

ODE Model Examples

Extinction



Recovery



$B = .01, \delta = .1, \mu = 10, K = 300$

Stochastic Model

Rates from the deterministic model become probabilities in the stochastic model:

$$\frac{df}{dt} = 1/2\, fm\, \mathrm{B}\, L - \delta\, f$$

$$\frac{dm}{dt} = (1/2\, fm + 1/2\, r\, m + f\, s)\, \mathrm{B}\, L - \delta\, m$$

$$\frac{ds}{dt} = (1/2\, r\, m + r\, s)\, \mathrm{B}\, L - \delta\, s$$

$$\frac{dr}{dt} = \mu - \delta\, r$$

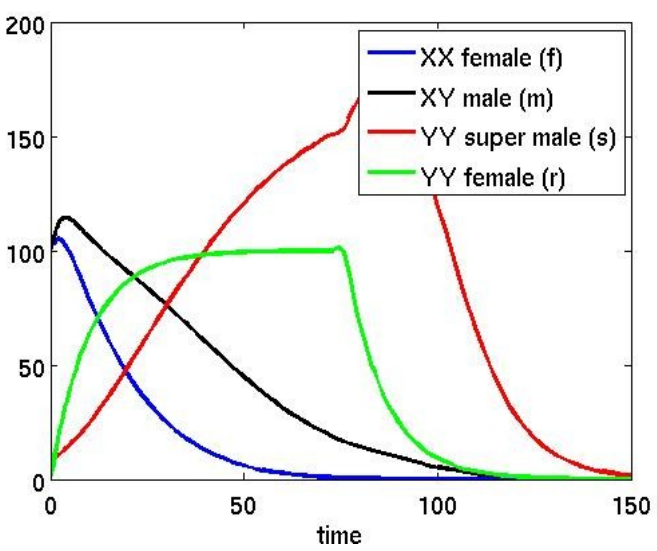
- Gillespie continuous time model
- *7 possible events:*

  3 birth events (f,m,s),
  4 death events (f,m,s,r)
- Stochastic rates:

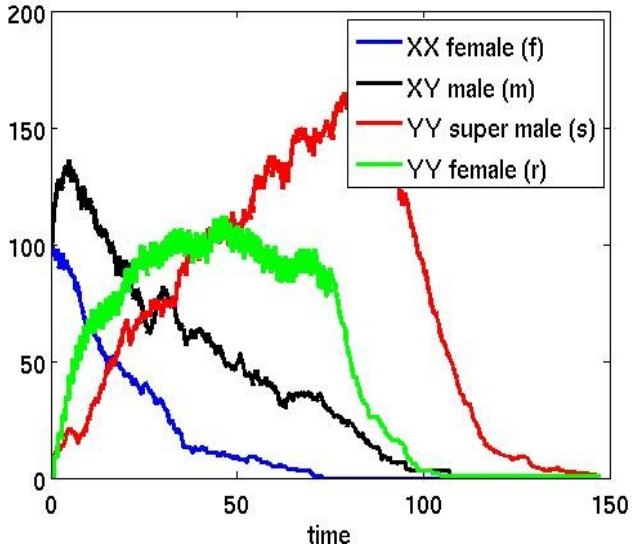
Event	Rate
f birth	$\frac{1}{2} fm\beta L$
m birth	$(\frac{1}{2} fm + \frac{1}{2} rm + fs)\beta L$
s birth	$(\frac{1}{2} rm + rs)\beta L$
f death	δf
m death	δm
s death	δs
r death	δr

Comparing the ODE and Stochastic Models

ODE Model



Stochastic Model

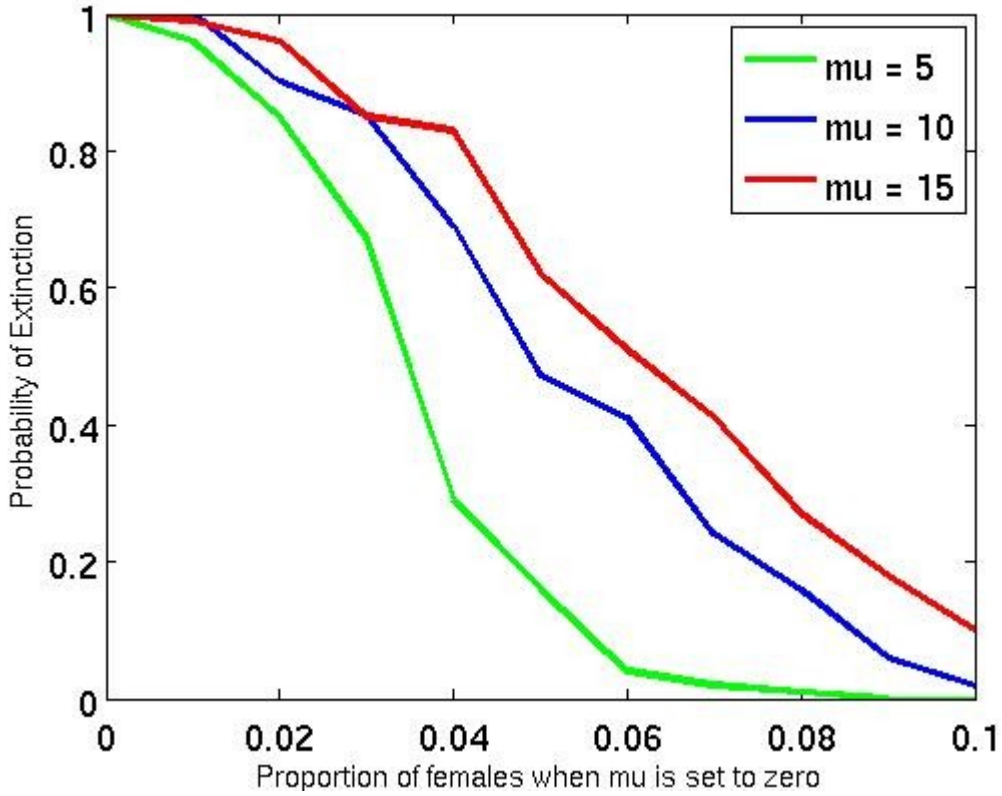


$B = .01, \delta = .1, K = 300$

Before time $= 75$, $\mu = 10$
After time $= 75$, $\mu = 0$

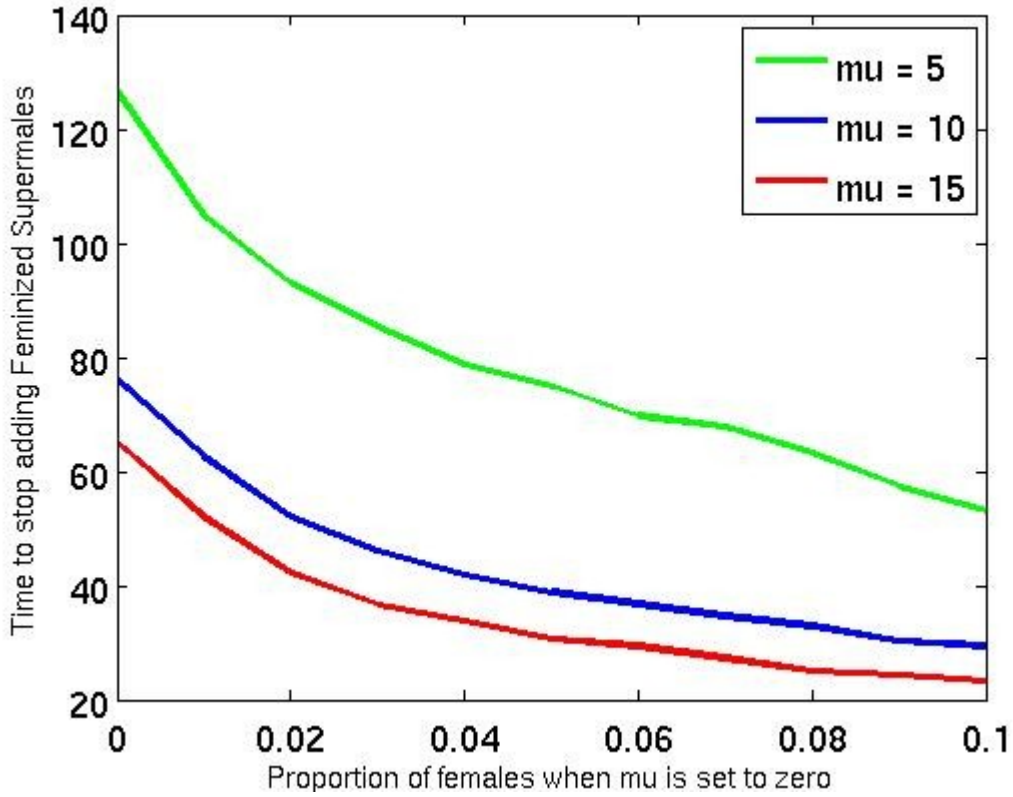
Stochastic Model Graphs

Probability of extinction (f=0), given that μ is set to 0 when f is a certain proportion of the total population:



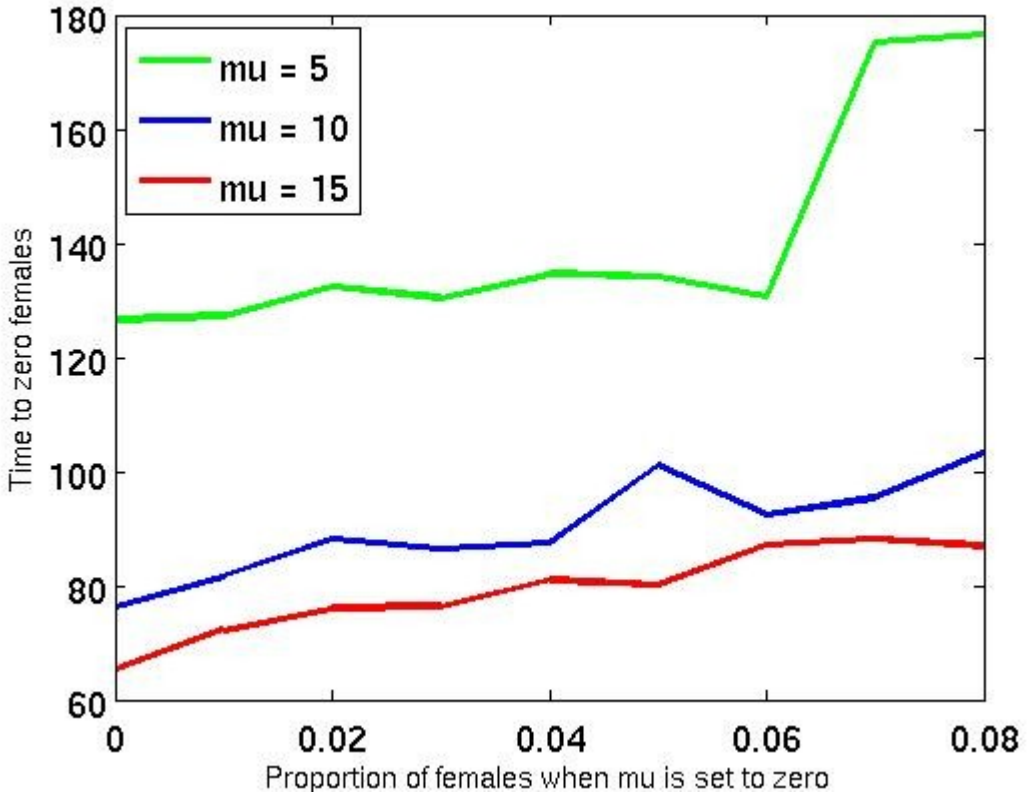
Stochastic Model Graphs (cont'd)

Average time that μ is set to 0, given a certain f proportion



Stochastic Model Graphs (cont'd)

Average time to extinction, given a certain f proportion



Spatial Model

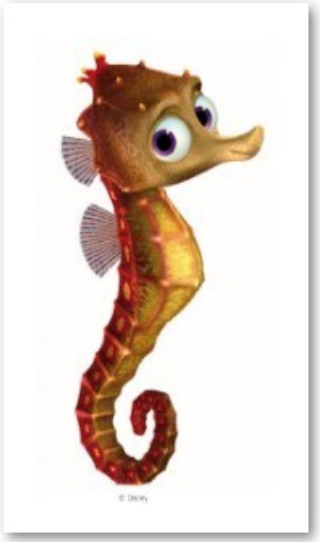
- Diffusive migration model
- Discrete time
- 24 events instead of 7 (16 migration, 7 birth/death, and 1 status quo)

3x3 grid example

(1,1) **4f, 5m, 2s, 1r**	**3f, 3m, 3s, 2r**	
5f, 7m, 1s, 1r		

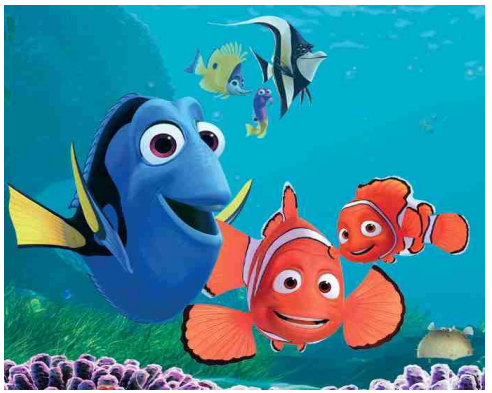
Future Work

- Adding feminized supermales to multiple grid cells
- Allowing the addition of feminized supermales to surpass carrying capacity
- Allowing migration events to surpass carrying capacity
- Continuous time spatial model



Acknowledgments

Thank you to Dr. May Boggess, Dr. Jay Walton, and Dr. Xueying Wang



Sources:

- Juan B. Gutierrez and John L. Teem, *A model describing the effect of sex-reversed YY fish in an established wild population: the use of a Trojan Y chromosome to cause extinction of an introduced exotic species,* Journal of Theoretical Biology, 241 (2006), 333-341.
- M.A. Hurley, P. Matthiessen, and A.D. Pickering, *A model for environmental sex reversal,* Journal of Theoretical Biology, 227 (2004), 159-165.
- Rana D. Parshad and Juan B. Gutierrez, *On the global attractor of the Trojan Y chromosome model,* Communications on Pure and Applied Analysis, 10 (2010), 339-359.