

# 2016 Texas A&M REU Miniconference

July 18–19, Blocker Building, Room 457

## SCHEDULE

MONDAY, July 18

08:00–08:25	Breakfast snacks in Blocker 246	Bluebaker
08:30–08:50	Higher-Dimensional Analogues of the Combinatorial Nullstellensatz	Jake D. Mundo
08:55–09:15	Bounding the Number of Distinct $p$ -adic Valuations of Integer Roots of Certain SPS-Polynomials	Kayla S. Cummings
09:20–09:40	Counting the $p$ -adic valuations of the roots of multivariate systems of polynomials	Cory Saunders
09:50–10:15	Pictures in Garden in Mitchell Bldg.	Chris Jarvis
10:25–10:45	Faltings Heights of CM Elliptic Curves and Special Gamma Values I	Tyler Genao
10:50–11:10	Faltings Heights of CM Elliptic Curves and Special Gamma Values II	Lindsay Cadwallader
11:15–11:35	Faltings Heights of CM Elliptic Curves and Special Gamma Values III	Olivia Cannon
11:40–12:00	On the Zeroes of Half-Integral Weight Eisenstein Series on $\Gamma_0(4)$	Samantha Moore
12:05–13:05	Lunch in Blocker 246	Taz
13:10–13:20	Slide Show!	Mitchell Eithun
13:20–13:40	Groebner Bases and the Neural Ideal	Jessica Liu
13:45–14:05	Neural Codes: Convexity and Computability	Aaron Chen

TUESDAY, July 19

09:30–09:50	A Complexity Bound for Real Zero Sets of $n$ -variate $(n + 4)$ -nomials	Lucy Yang
09:55–10:15	On the Topology of Real Zero Sets of Bivariate Pentanomials	Erin Lipman
10:20–10:40	Moments of $L$ -functions associated to Newforms of Squarefree Level	David Burt
10:40–10:55	Break!	
10:55–11:15	Analyzing Methods to Determine Pairwise Correlations Between Neurons	Luna Bozeman
11:20–11:40	Stability of Control System of Intracellular Iron Homeostasis: A Mathematical Proof	Adriana Morales
11:45–12:05	Proving Global Stability of Processive Phosphorylation Systems	Mitchell Eithun
12:10–	Lunch in Blocker 246	Polly's

As always, we gratefully acknowledge the support of our Department Head Emil Straube, Associate Department Head Paulo Lima-Filho, Undergraduate Program Assistant Donna Hoffman, and the support of the National Science Foundation through REU grant DMS-1460766.

# ABSTRACTS

(In order of appearance)

## Higher-Dimensional Analogues of the Combinatorial Nullstellensatz

by Jake D. Mundo

The celebrated Combinatorial Nullstellensatz of Alon describes the form of a polynomial which vanishes entirely on a Cartesian product of one dimensional sets. We explore analogues of the Combinatorial Nullstellensatz in higher dimensions, that is, we describe the form of polynomials which vanish entirely on Cartesian products of arbitrary dimensional sets, with a stronger theorem for the special case where all the sets are two-dimensional. We also discuss possible applications of these results to similar generalizations of the famous Schwartz-Zippel lemma, which bounds the amount of intersection between a variety and a Cartesian product of one-dimensional sets.

## Bounding the Number of Distinct $p$ -adic Valuations of Integer Roots of Certain SPS-Polynomials

by Kayla S. Cummings

An SPS-polynomial is a polynomial expressible as a sum of products of sparse univariate polynomials. SPS-polynomials are closely related to depth-4 arithmetic circuits (of recent interest in complexity theory), and Koiran has shown earlier that new lower bounds for the complexity of the permanent hold if SPS-polynomials of low complexity have few integer roots. Some effort has been made toward bounding the number of real roots of SPS-polynomials, but bounding the number of integer roots still appears out of reach. Bounding  $p$ -adic valuations of the integer roots is a potentially promising, alternative approach that has yet to be explored. We show that an upper bound for the number of  $p$ -adic valuations, in line with Koiran's conjectures, can be proven for a particular family of SPS-polynomials.

## Counting the $p$ -adic valuations of the roots of multivariate systems of polynomials

by Cory Saunders

Recent discoveries by Koiran, Portier, and Rojas have shown that finding sufficiently good upper bounds on the number of integer roots of certain multivariate systems of polynomials have deep implications on complexity theory. While explicitly finding bounds on the roots is difficult, we may draw techniques from  $p$ -adic tropical geometry to help find upper bounds on the number of valuations of the roots of the system. In this talk, we present techniques on how to bound the number of  $p$ -adic valuations of the roots for a certain system of polynomials. We can reduce this problem to looking at the intersections of a specific arrangement of hyperplanes.

## Faltings Heights of CM Elliptic Curves and Special Gamma Values I, II, and III

respectively by Tyler Genao, Lindsay Cadwallader, Olivia Cannon

In this talk, we will show how the Faltings height of an elliptic curve over  $Q$  with complex multiplication by an order in an imaginary quadratic field can be explicitly evaluated in terms of values of Euler's Gamma function at rational numbers.

## On the Zeroes of Half-Integral Weight Eisenstein Series on $\Gamma_0(4)$

by Samantha Moore

We locate all but  $O(\sqrt{k \log k})$  zeroes of the half integral weight Eisenstein series  $E_\infty(z, k)$  of  $\Gamma_0(4)$  for  $k$  sufficiently large. To do this, we relate  $E_\infty(z, k)$  to  $\Gamma_0(4)$ 's other Eisenstein series,  $E_0(z, k)$  and  $E_{\frac{1}{2}}(z, k)$ , which are easier to study in the regions of which zeroes occur. We will use trigonometric approximations of  $E_0(z, k)$  and  $E_{\frac{1}{2}}(z, k)$  in order to locate the zeroes.

## Groebner Bases and the Neural Ideal

by Jessica Liu

TBA

**Neural Codes: Convexity and Computability**

by Aaron Chen

TBA

**A Complexity Bound for Real Zero Sets of  $n$ -variate  $(n + 4)$ -nomials**

by Lucy Yang

TBA

**On the Topology of Real Zero Sets of Bivariate Pentanomial**

by Erin Lipman

TBA

**Moments of  $L$ -functions associated to Newforms of Squarefree Level**

by David Burt

Moments of  $L$ -functions provide a powerful tool for studying the analytical properties of these families of functions within the critical strip. For  $L$ -functions associated to newforms of arbitrary even integer weight, we obtain explicit asymptotic formulae for the first two shifted moments as the level of the  $L$ -function goes to infinity over squarefree integers. This generalizes work of Duke, who obtained a similar result for prime level and weight 2, and Akbary, who obtained a similar result in the case of prime level and arbitrary weight. The main ingredient in these calculations will be an approximate version of the trace formula for newforms established in recent work of Petrow and Young”

**Analyzing Methods to Determine Pairwise Correlations Between Neurons**

by Luna Bozeman

TBA

**Stability of Control System of Intracellular Iron Homeostasis: A Mathematical Proof**

by Adriana Morales

TBA

**Proving Global Stability of Processive Phosphorylation Systems**

by Mitchell Eithun

TBA