An Application of Compressive Sensing to Image and Video Compression

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Introduction

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- The field was pioneered by Candés, Romberg, Tao [1][2][3][4] and Donoho [5].
- Compressive Sensing works in a 'naive' manner, requiring no prior knowledge of the signal and instead relying on the structure that are often found in linearly-modeled signals.

Basic Idea

Suppose $\mathbf{x} \in \mathbb{R}^n$ is our signal that we are interested in compressing. We perform the compression by multiplying $\mathbf{x}$ by Φ, an $m \times n$-matrix, where $m \ll n$.

$$\mathbf{y} = \Phi \mathbf{x} \tag{1}$$

Thus $\mathbf{y}$ represents our compressed signal.
By imposing conditions on $\mathbf{x}$ and Φ, we can recover our signal.

The Restricted Isometry Property

A signal can be recovered if there exists a $\delta_K \in (0, 1)$, where the Φ matrix satisfies

$$(1 - \delta_K)\|\mathbf{x}\|_2^2 \leq \|\Phi\mathbf{x}\|_2^2 \leq (1 + \delta_K)\|\mathbf{x}\|_2^2. \tag{2}$$

where $\mathbf{x} \in \Sigma_K = \{\mathbf{x} : \|\mathbf{x}\|_0 \leq K\}$, $\| \cdot \|$ denoting the sparisty of the vector, the number of nonzero entires. This property is known as the *Restricted Isometry Property (RIP)*.

The Restricted Isometry Property Cont.

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The Restricted Isometry Property Cont.

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- Construct *Phi* by choosing the entries from a Normal distribution with zero mean and a standard deviation of m^{-1}.
- Construct *Phi* by randomly choosing m distinct rows of a wavelet matrix.

Related Theorems

Theorem

[6] Let Φ be an $m \times n$-matrix that satisfies the RIP of order $2K$ with constant $\delta \in (0, \frac{1}{2}$. Then

$$m \geq C \log\left(\frac{N}{K}\right) \tag{3}$$

where $C = (2\log(\sqrt{24}+1)^{-1}$.

Related Theorems Cont.

Theorem

[6] If

$$K < \frac{1}{2}\left(1 + \frac{1}{\mu(\Phi)}\right) \text{ where } \mu(\Phi) = \max_{1 \le i < j \le n} \frac{|\langle \phi_i, \phi_j \rangle|}{\|\phi_i\|_2 \|\phi_j\|_2} \tag{4}$$

then for each measurement vector $\mathbf{y} \in \mathbb{R}^m$ *there exists at most one signal* $\mathbf{x} \in \Sigma_K$ *such that* $\mathbf{y} = \Phi\mathbf{x}$.

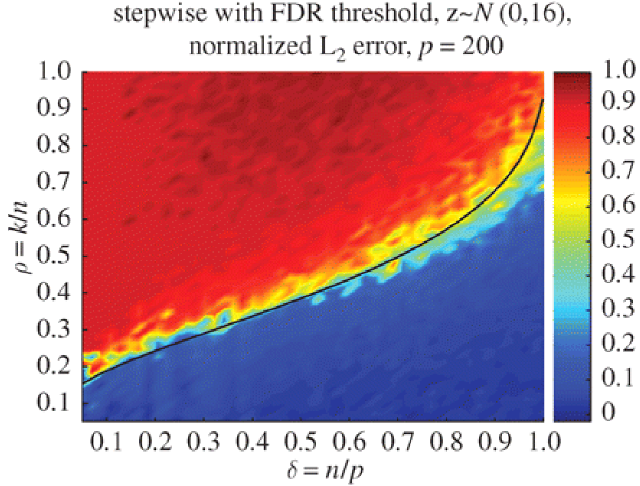
Reconstruction

- To recover our original signal, we solve the convex optimization problem

$$\min_{\mathbf{x}} \|\mathbf{y} - \Phi\mathbf{x}\|_2 + \|\mathbf{x}\|_1 \tag{5}$$

- Algorithms such as linear programming and gradient descent can be used.
- The algorithm we use is called the *Multihypothesis Block-based Compressive Sensing*.

Donoho-Tanner Phase Transition [7]



Design of our Algorithm: The Compression

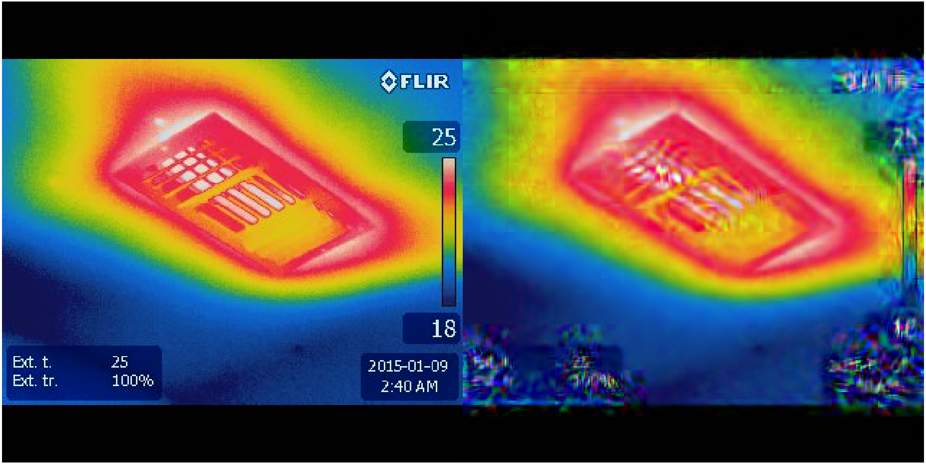
- A satellite takes a picture while in flight.
- The image is then separated into a red, green, and blue channels.
- Each Channel is then taken and multiplied by a different Φ Matrix.

Design of our Algorithm: The Reconstruction

- The picture is then received on Earth.
- Each individual channel is reconstructed by parallel computing using a cluster of computers.
- After each channel is reconstructed the channels are combined back into one picture.

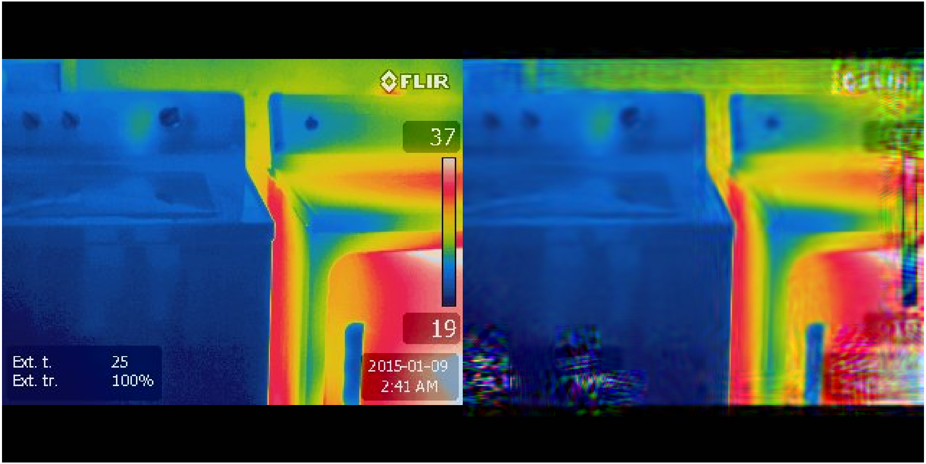
Reconstructed Images

320x320



Reconstructed Images Cont.

320x320



Reconstructed Images Cont.

600x600

Reconstructed Images Cont.

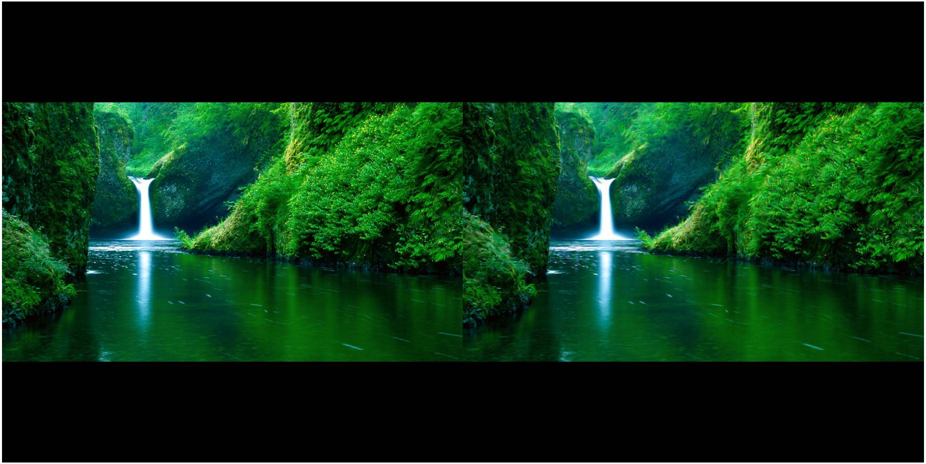
1024 x1024

Reconstructed Images Cont.

1024 x1024

Reconstructed Images Cont.

1920 x1920



Design of our Algorithm: Video

- The video is taken and broken up into frames.
- Each frame is treated as image and compressed then reconstructed using the same procedure as in the previous two slides.
- The main difference is that in this code, after the 1^{st} frame, the previous frame is used as an initial guess.

Bibliography I

E. Candés, J. Romberg, and T.Tao,
Stable signal recovery from incomplete and inaccurate information,
Commun. Pure Appl. Math, vol.59, pp.1207-1233, 2005.

E. Candés, T.Tao,
The Dantzig selector. Statistical estimation when p is much larger than n,
Ann. Statist. 2007 [Online]. Available:
http://arxiv.org/abs/math.ST/0506081.

E. Candés, T.Tao,
Near optimal signal recovery from random projections: Universal encoding strategies,
IEEE Trans. Inform. Theory, vol.52, pp.5406-5425, Dec. 2006.

Bibliography II

E. Candés, J. Romberg, and T.Tao,
Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information,
IEEE Trans. Inform. Theory, vol.52, pp.489-509, Feb. 2006.

D. Donoho,
Compressed Sensing,
IEEE Trans. Inform. Theory, vol.52, pp. 1289-1306, Apr. 2006.

Richard Baraniuk, Mark A. Davenport, Marco F. Duarte, Chinmay Hegde, Jason Laska, Mona Sheikh, Wotao Yin,
An Introduction to Compressive Sensing,
Connexions, Rice University, Houston, Texas, Online:
$< http://cnx.org/content/col11133/1.5/ >$.

Bibliography III

David Donoho, Jared Tanner,
Observed Universality of Phase Transitions in High-Dimensional Geometry, with Implications for Modern Data Analysis and Signal Processing.