Homework 14

Math 171H (section 201), Fall 2023

This homework is due on **Tuesday, November 21** at the start of class. (Turn in answers to questions 1–10.)

0. Read Sections 4.9, 5.2, 5.3

1. Determine the most general antiderivatives of the following functions:
   (a) $f(x) = \frac{1}{3} - \frac{2}{x}$
   (b) $f(x) = 2^x + e^{3x} + x\sqrt{x}$
   (c) $f(x) = -2\sin x$

2. Find a function $f(x)$ for which
   - $f'(x) = x^3$, and
   - the line $y = x + 1.25$ is tangent to the graph of $f(x)$.

3. Sketch a graph of $F(x) = \int_1^x f(x)dx$, where

$$f(x) = \begin{cases} 1 & \text{if } x \leq 2 \\ 2 & \text{if } x > 2 \end{cases}$$

4. (a) ~~Sketch the region under the curve $y = \sqrt{x}$, for $0 \leq x \leq 16$. Compute the area.~~
   (b) ~~Sketch the region bounded by $y = x^2$ and $y = 18 - x^2$. Compute the area.~~
   (c) ~~Compute $\int_0^1 (1+s)^3 ds$.~~

5. Prove or disprove:

$$\int_a^b f(x)dx \;=\; \int_{a+c}^{b+c} f(x-c)dx$$

6. Prove or disprove:
   (a) If $\int_a^b f(x)dx = 0$, then $f(x) = 0$ for all x in $[a, b]$.
   (b) If $\int_a^b f(x)dx = 0$ and $f(x) \geq 0$ for all x in $[a, b]$, then $f(x) = 0$ for all x in $[a, b]$.
   (c) If $f(x)$ is continuous and non-negative ($f(x) \geq 0$) on $[a, b]$, and $f(x_0) > 0$ for some x_0 in (a, b), then $\int_a^b f(x)dx > 0$.

7. Assume a and b are real numbers. Compute $F'(x)$.
   (a) $F(x) = \int_0^x x^2 f(t)dt$

(b) $F(x) = \int_a^{x^2} \cos^3(t)dt$

(c) $F(x) = \int_2^{\left(\int_1^x \ln s\ ds\right)} \sqrt{t}dt$

(d) $F(x) = \int_a^x \left(\int_b^y 3^t dt\right) dy.$

8. (Write your own problem!) A $\underset{\text{vehicle}}{\underline{\hspace{4cm}}}$ is traveling at $\underset{\text{positive number}}{\underline{\hspace{4cm}}}$ miles/hour when the brakes are applied, producing a constant deceleration of $\underset{\text{positive number}}{\underline{\hspace{4cm}}}$ feet/sec^2. What is the distance traveled before the vehicle comes to a stop?

9. (Write your own problem!) Give an example of a definite integral of a non-constant function, for which the Riemann-sum approximation by $\underset{\text{positive number, at least 4}}{\underline{\hspace{4cm}}}$ rectangles and left endpoints is equal to $\underset{\text{number}}{\underline{\hspace{4cm}}}$.

10. Compute the definite integral $\int_1^3 (3-x)dx$ in three ways:

(a) by drawing the graph, and computing the appropriate area.

(b) using the limit definition (via Riemann sums).

(c) ~~using the Fundamental Theorem of Calculus.~~