Homework 8

Math 171H (section 201), Fall 2023

This homework is due on **Thursday, October 12** at the start of class. (Turn in answers to questions 1–7.)

0. Read Sections 3.5–3.6 and 13.1–13.2

1. An ant is walking around the unit circle. Let $\theta(t)$ denote the angle of the ant at time t. Let $(x(t), y(t))$ denote the position of the ant at time t.

   (a) State an expression for $x'(t)$ in terms of $\theta(t)$.

   (b) State an expression for the second derivative $y''(t)$ in terms of $\theta(t)$.

2. Compute the derivatives of the following functions:

   (a) $\arccos \sqrt{x}$

   (b) $\ln(-1/x)$

   (c) $\frac{1}{\ln x}$

   (d) $x^{\cos x}$

   (e) 23^{-x}

3. Compute $f'(x)$ in terms of $g'(x)$, when f and g are related as follows:

   (a) $f(x) = g(-x)$

   (b) $f(x) = -g(x)$

   (c) $f(x) = x \cdot g(2x)$

4. Consider the circle defined by $x^2 + y^2 = 5$

   (a) Find all points on the circle with y-value equal to 2.

   (b) For each of these points, compute the slope of the tangent line at that point.

5. Consider the vector-valued function $\underline{\mathbf{r}}(t) = \langle t^3 - 3t^2, \ t^3 - 3t \rangle$.

   (a) Compute $\underline{\mathbf{r}}'(-2)$

   (b) Find all points on the curve defined by $\underline{\mathbf{r}}(t)$ at which the tangent is horizontal.

   (c) Find all points on the curve defined by $\underline{\mathbf{r}}(t)$ at which the tangent is vertical.

6. Prove the following: *If f is a differentiable, one-to-one function, and $f'(f^{-1}(a)) = 0$, then the inverse function f^{-1} is NOT differentiable at $x = a$.*

7. Consider a vector-valued function $\underline{\mathbf{r}}(t) = \langle x(t), y(t) \rangle$. Let a be a real number. Prove that if $\lim_{t \to a} x(t)$ and $\lim_{t \to a} y(t)$ exist, then

$$\lim_{t \to a} \underline{\mathbf{r}}(t) \ = \ \left\langle \ \lim_{t \to a} x(t), \ \lim_{t \to a} y(t) \ \right\rangle .$$