Homework 8

Math 220 (section 906), Fall 2018

This homework is due on Thursday, October 18. (Turn in your answers to questions 1–8.) You may cite results from class, as appropriate.

0. *(This problem is not to be turned in.)* Read Sections 5.1–5.3.

   (a) Explain what is wrong with the following: *Consider a function $f : \mathbb{Z} \to 9$.*

   (b) Explain what is wrong with the following: *Consider a function $f : \mathbb{Z} \mapsto \mathbb{R}$.*

   (c) Give an example of a function $f : \mathbb{Z} \to \mathbb{R}$.

   (d) Give an example of a function $f : \mathbb{R} \to \mathbb{Q}$.

1. Determine whether each of the following sets is the graph of some function. Prove your answers.

   (a) $\{(x, y) \in \mathbb{R}^2 \mid x = y^2\}$

   (b) $\{(x, y) \in \mathbb{Z}^2 \mid x - y - 5\}$

2. (No proofs necessary for this problem)

   (a) List *all* functions $f : \mathbb{Z} \to \{8\}$ (functions with domain $\mathbb{Z}$ and codomain $\{8\}$).

   (b) List *all* **injective** (one-to-one) functions $f : \{0, 1\} \to \{2, 3, 4\}$.

   (c) List *all* **surjective** (onto) functions $f : \{0, 1\} \to \{2, 3\}$.

3. Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(n) = 2n$ if n is even and $f(n) = n - 3$ if n is odd.

   (a) *Prove or disprove*: f is injective.

   (b) *Prove or disprove*: f is surjective.

4. Let $f : A \to C$ and $g : B \to D$ be functions. Consider the following function:

$$h : A \times B \to C \times D$$
$$(a, b) \mapsto (f(a), g(b)) .$$

   (a) *Prove or disprove*: If f and g are injective, then so is h.

   (b) *Prove or disprove*: If f and g are surjective, then so is h.

5. Let A be a nonempty set. Assume $b \notin A$. Consider the following function:

$$h : \mathcal{P}(A) \to \mathcal{P}(A \cup \{b\})$$
$$S \mapsto S \cup \{b\} .$$

   (a) *Prove or disprove*: h is injective.

   (b) *Prove or disprove*: h is surjective.

   (c) Is h bijective? Explain your answer.

6. Section 5.1 #2, 6

7. Section 5.2 #1, 2

8. Section 5.3 #3