

# Homework 9

Math 220 (section 906), Fall 2018

This homework is due on Thursday, October 25. (Turn in your answers to questions 1–9.) You may cite results from class, as appropriate.

0. (*This problem is not to be turned in.*) Read Sections 5.4 and 5.5.
  - (a) How does the book's definition of *invertible* differ from the one we gave in class? (See Definition 5.4.1 and Proposition 5.4.3.)
  - (b) What is the difference between  $f(x)$  and  $f[X]$ ?
1.
  - (a) Suggest an exam problem pertaining to sets.
  - (b) Suggest an exam problem pertaining to functions.
2. Consider the following function:

$$f : \mathbb{R} \setminus \left\{ \frac{3}{5} \right\} \rightarrow \mathbb{R} \setminus \left\{ \frac{2}{5} \right\}$$
$$x \mapsto \frac{2x + 1}{5x - 3} .$$

Is it invertible? Is it bijective? Prove your answers.

3. Is the following function  $f : \mathbb{R} \rightarrow \mathbb{R}$  invertible? If so, find the inverse function:

$$f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 . \end{cases}$$

4. Assume that  $a, b, c, d$  are real numbers with  $a \neq 0$  and  $c \neq 0$ . Let  $f(x) = ax + b$  and  $g(x) = cx + d$  be functions (with both domain and codomain equal to  $\mathbb{R}$ ). Is the function  $h := g \circ (f^{-1})$  invertible? If so, find the inverse function of  $h$ .
5. Let  $f : A \rightarrow B$  be a function. Prove or disprove the following:
  - (a) If  $f$  is surjective, then there exists a function  $h : B \rightarrow A$  such that  $h \circ f = Id_A$ .
  - (b) If  $f$  is injective, then there exists a function  $h : B \rightarrow A$  such that  $h \circ f = Id_A$ .
  - (c) If  $f$  is surjective, then there exists a function  $h : B \rightarrow A$  such that  $f \circ h = Id_B$ .
  - (d) If  $f$  is injective, then there exists a function  $h : B \rightarrow A$  such that  $f \circ h = Id_B$ .
6. Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions. Prove or disprove the following:
  - (a) If  $g \circ f$  is surjective, then  $f$  is surjective.
  - (b) If  $g \circ f$  is surjective, then  $g$  is surjective.
  - (c) If  $g \circ f$  is injective, then  $f$  is injective.
  - (d) If  $g \circ f$  is injective, then  $g$  is injective.
7. For each of the statements in #6, state the converse, and then prove or disprove it.
8. Section 5.4 #1(m)
9. Section 5.5 #1, 2, 4, 6, 10

## Writing Assignment 5

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This homework is due on **Thursday, October 25**.

*This week, you will critique another student's draft. Staple your comments to the draft, and make sure both your name and the author's name appear on the front. Do not staple this to the rest of your homework.*

- (a) Read through the draft. Mark each spot you got confused.
  - (b) What questions do you have for the authors?
  - (c) What aspects of the draft did you like?
  - (d) What could the author do to improve this draft?
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## Writing Assignment 6

This homework is due on **Thursday, November 1**.

- Extend your draft to include four sections of your final paper:
  1. the introduction (what will your paper be about?),
  2. the mathematical background (define and/or explain all unfamiliar terms), and
  3. *two* sections developing main ideas from Writing Assignment 3.
- *Edit your draft in response to all comments (to be given next week).*
- The required length is at least four pages.
- *Please print 1 copy of your draft.*
- If you do **not** turn in this draft, you will receive a 5% penalty on the final paper.