Homework 13

Math 300, Fall 2022

This homework is due on Friday, Nov. 18.

0. *(This problem is not to be turned in.)* Read Sections 8.2–8.3.
   (a) Section 8.1 #7, 11
   (b) Section 8.2 #5, 13
   (c) Section 8.3 #2
1. Read the excerpt from *Becoming an academic writer* (Goodson) on tightening paragraphs, provided in class (and available at the TAMU library). Pick one of the paragraphs in your final paper draft, and edit it accordingly (as a reminder, two copies of your draft are due Mon., Nov. 22). Write a few sentences reflecting on the process (did anything in the excerpt surprise you? what edits did you make to your paragraph? any changes you plan to make in your writing, going forward?).
2. Prove or disprove: *If $A_1, A_2, \ldots$ are finite sets, then their union $\bigcup_{i=1}^{\infty} A_i$ is also finite.*
3. Prove the following:
   (*Hint*: These results are related to Theorems 8.3.3 and 8.3.7 in the textbook, but we didn't prove them in class. So, if you use them, please prove them!)
   (a) If A and B are countable sets, then $A \cup B$ is countable.
   (b) If A and B are countable sets, then $A \times B$ is countable.
4. Consider the statement, *For every nonnegative integer n, if A is a finite set of cardinality n, then the number of subsets of A is 2^n.*
   (a) State the **base case** for a proof (of the statement) by induction (on n).
   (b) Prove the base case.
   (c) State the **inductive hypothesis.**
   (d) State the *goal* of the **inductive step**.
   (e) To complete the inductive step, let A be a set of cardinality $k+1$, which we write as $A = \{x_1, x_2, \ldots, x_{k+1}\}$. Consider the set $B = \{x_1, x_2, \ldots, x_k\}$ (which is a subset of A). How are the subsets of A that do NOT contain the element x_{k+1} related to the subsets of B? Explain.
   (f) How are the subsets of A that contain x_{k+1} related to the subsets of B? Explain.
   (g) Use your answers to (e) and (f) – plus the inductive hypothesis – to count the total number of subsets of A.
   (h) Do your above answers prove that $|\mathcal{P}(A)| = 2^{|A|}$ for every finite set A? (Compare with Theorem 8.2.9 in your textbook.) Explain.
5. Section 8.2 #6, 12
6. Section 8.3 #17 (*Hint*: Use #2.)