Homework 14

Math 300, Fall 2022

This homework is due on WEDNESDAY, Nov. 30.

0. *(This problem is not to be turned in.)* Read Sections 6.1–6.3
   (a) Are the Well-ordering principle and the Principle of mathematical induction, equivalent?
   (b) What does it mean that the Well-ordering principle is an *axiom*?
   (c) What is a *linear combination*? (page 138)
   (d) Prove the following:
   If a is a nonzero integer and c is an integer, then $\gcd(a, ca) = a$.
1. Read about "that" vs. "which". How should you decide which one to use (in your writing)?
2. Answer the following questions, and explain your answers.
   (a) Is the Well-ordering principle still true if, instead of subsets of the *nonnegative* integers, we consider subsets of the *positive* integers?
   (b) Does every non-empty subset of $\mathbb{Z}$ have a *largest* element?
   (c) Does every non-empty subset of $\mathbb{R}$ have a *smallest* element?
3. Consider the function $f : \mathbb{Z} \times \mathbb{N} \to \mathbb{Z} \times \mathbb{Z}$ (where $\mathbb{N} := \{1, 2, 3, \dots\}$) given by:
   $$f(a, b) \;=\; (q, r) \;,$$
   where q and r are the "quotient" and "remainder" obtained by applying the division algorithm to a and b (so, $a = bq + r$).
   (a) Compute $f(40, 7)$ and $f(-20, 3)$. (No proof necessary.)
   (b) Is f one-to-one? Prove your answer.
   (c) Is f onto? Prove your answer.
4. (No proofs necessary for this problem.)
   (a) Give an example of integers a and b, with $a < 0$ and $b < 0$, such that their g.c.d. is 8.
   (b) Give an example of integers a and b, with $a \geq 20$ and $b \geq 20$, such that their g.c.d. is 12.
5. Let a and b be nonzero integers. Let $d = \gcd(a, b)$. Prove that a/d and b/d are both integers.
6. Section 6.1 #1, 3 (*Hint:* Read Example 6.1.4.)
7. Section 6.2 #1(a, b), 3
8. Section 6.3 # 3