Homework 5

Math 300, Fall 2022

This homework is due on Friday, September 23. (Turn in your answers to questions 1–2.)

0. *(This problem is not to be turned in.)* Read Sections 2.2 and 2.3
   (a) Section 2.3 #2
   (b) What is the difference between a proof by contradiction and a proof by contrapositive?
   (c) What is the Fundamental Theorem of Algebra?
   (d) Prove that an integer n is even if and only if $-n$ is even.
   (e) Conclude (explain why you can!) that an integer n is odd if and only if $-n$ is odd.
   (f) Prove that an integer n is even if and only if its last digit (the ones digit) is 0, 2, 4, 6, or 8. (*Hint*: For $n > 0$, consider the remainder after dividing by 10; for $n < 0$, use a previous problem.)
   (g) Conclude (explain why you can!) that an integer n is odd if and only if the last digit is 1, 3, 5, 7, or 9.
1. Section 2.2 #3, 9
2. Prove or disprove the following claims:
   (a) There is a smallest positive integer.
   (b) There is a smallest rational number.
   (c) There is a largest real number.
   (d) $\sqrt[3]{2}$ is irrational.